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Bioperation-somewhat continuous functions

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Abstract In this paper, we introduce and study the weak form of $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions called somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions between bioperation-topological spaces.

Key Words $\gamma\vee\gamma'$-semiopen set, $\gamma\vee\gamma'$-somewhat semicontinuous functions

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1 Introduction and Preliminaries

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, seperation axioms etc. by utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces. Ogata and Maki [3] introduced the notion of $\tau_{\gamma\vee\gamma'}$, which is the collection of all $\gamma\vee\gamma'$-open sets in a bioperation-topological space (X,τ,γ,γ'). In this paper, we introduce and study the weak form of $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions called somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions between bioperation-topological spaces.

2 Preiliminaries

The closure and the interior of a subset A of (X,τ) are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

Definition 2.1. *[1] Let (X,τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $\mathcal{P}(X)$ of X such that $V\subset V^{\gamma}$ for each $V\in\tau$, where V^{γ} denotes the value of τ at V. It is denoted by $\gamma:\tau\to\mathcal{P}(X)$.*

Definition 2.2. *[3] A topological space (X,τ) equipped with two operations namely γ and γ' defined on τ is called a bioperation-topological space and it is denoted by (X,τ,γ,γ').*

Definition 2.3. *A subset A of a bioperation-topological space (X,τ,γ,γ') is said to be $\gamma\vee\gamma'$-open set [3] if for each $x\in A$ there exists an open neighbourhood U of x such that $U^{\gamma}\cup U^{\gamma'}\subset A$. The complement of $\gamma\vee\gamma'$-open set is called $\gamma\vee\gamma'$-closed. $\tau_{\gamma\vee\gamma'}$ denotes set of all $\gamma\vee\gamma'$-open sets in (X,τ).*

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Definition 2.4. *[3] For a subset A of (X,τ), $\tau_{\gamma\vee\gamma'}$-$\mathrm{Cl}(A)$ denotes the intersection of all $\gamma\vee\gamma'$-closed sets containing A, that is, $\tau_{\gamma\vee\gamma'}$-$\mathrm{Cl}(A)=\bigcap\{F : A\subset F, X\backslash F\in\tau_{\gamma\vee\gamma'}\}$.*

Definition 2.5. *Let A be any subset of X. The $\tau_{\gamma\vee\gamma'}$-$\mathrm{Int}(A)$ is defined as $\tau_{\gamma\vee\gamma'}$-$\mathrm{Int}(A)=\cup\{U : U$ is a $\gamma\vee\gamma'$-open set and $U\subset A\}$.*

Definition 2.6. *A subset A of a topological space (X,τ) is said to be $\gamma\vee\gamma'$-semiopen [2] if $A\subset\tau_{\gamma\vee\gamma'}$-$\mathrm{Cl}(\tau_{\gamma\vee\gamma'}$-$\mathrm{Int}(A))$.*

Theorem 2.7. *A subset A of a bioperation-topological space (X,τ,γ,γ') is $\gamma\vee\gamma'$-semiopen if,and only if for each $x\in X$ there exists a $\gamma\vee\gamma'$-semiopen set U suchthat $x\in U\subset A$.*

Definition 2.8. *[2] A function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is said to be $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous if the inverse image of every $\beta\vee\beta'$-open set in (Y,σ,β,β') is a $\gamma\vee\gamma'$-semiopen set in (X,τ,γ,γ').*

3 Somewhat bioperation-semicontinuous functions

In this section, we introduce and study the weak form of $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions called somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous functions between bioperation-topological spaces.

Definition 3.1. *A function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is said to be somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous if for every $\beta\vee\beta'$-open set U such that $f^{-1}(U)\neq\emptyset$, there exists a $\gamma\vee\gamma'$-semiopen set V in X such that $V\neq\emptyset$ and $V\subset f^{-1}(U)$.*

It is clear that every $\gamma\vee\gamma'$-semicontinuous function is somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous but the converse is not true as shown by the following example

Example 3.2. *Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{b\},\{a,b\},X\}$;$\sigma=\{\emptyset,\{a\},\{c\},\{a,c\},X\}$. Let $\gamma,\gamma':\tau\to\mathcal{P}(X)$ and $\beta,\beta':\sigma\to\mathcal{P}(X)$ be operations defined as follows:*

$$A^{\gamma}=\begin{cases} A & \text{if } A=\{a\},\\ X & \text{otherwise,}\end{cases}\quad A^{\gamma'}=A,\quad for\ all\quad A\in\tau,$$

$$A^{\beta}=\begin{cases} A & \text{if } a\in A,\\ A\cup\{a\} & \text{if } a\notin A,\end{cases}\ \text{and}\ A^{\beta'}=\begin{cases} A & \text{if } c\in A,\\ A\cup\{c\} & \text{if } c\notin A.\end{cases}$$

Then the identity function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$ is somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous but not $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous.

Definition 3.3. *A subset M of a bioperation-topological space (X,τ,γ,γ') is said to be $\gamma\vee\gamma'$-semidense in X if there is no proper $\gamma\vee\gamma'$-semiclosed set C in X such that $M\subset C\subset X$.*

Theorem 3.4. *For a surjective function $f:(X,\tau,\gamma,\gamma')\to(Y,\sigma,\beta,\beta')$, the following statements are equivalent:*

1. *f is somewhat $\gamma\vee\gamma'$-$\beta\vee\beta'$-semicontinuous.*

2. *If C is a $\beta \vee \beta'$-closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper $\gamma \vee \gamma'$-semiclosed subset D of X such that $D \supset f^{-1}(C)$.*

3. *If A is a $\beta \vee \beta'$-semiopen subset of Y such that $f^{-1}(A) \neq X$, then there is a proper $\gamma \vee \gamma'$-semiopen subset B of X such that $f^{-1}(A) = B$;*

4. *If M is a $\gamma \vee \gamma'$-semidense subset of X, then $f(M)$ is a $\beta \vee \beta'$-dense subset of Y.*

Proof. (1) $\Rightarrow$ (2): Let C be a $\beta \vee \beta'$-closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y \backslash C$ is a $\beta \vee \beta'$-open set in Y such that $f^{-1}(Y \backslash C) = X \backslash f^{-1}(C) \neq \emptyset$. By (1), there exists a $\gamma \vee \gamma'$-semiopen set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(Y \backslash C) = X \backslash f^{-1}(C)$. This means that $X \backslash V \supset f^{-1}(C)$ and $X \backslash V = D$ is a proper $\gamma \vee \gamma'$-semiclosed set in X.
(2) $\Rightarrow$ (1): Let $U \in \sigma_{\beta \vee \beta'}$ and $f^{-1}(U) \neq \emptyset$. Then $Y \backslash U$ is $\beta \vee \beta'$-closed and $f^{-1}(Y \backslash U) = X \backslash f^{-1}(U) \neq X$. By (2), there exists a proper $\gamma \vee \gamma'$-semiclosed set D such that $D \supset f^{-1}(Y \backslash U)$. This implies that $X \backslash D \subset f^{-1}(U)$ and $X \backslash D$ is $\gamma \vee \gamma'$-semiopen and $X \backslash D \neq \emptyset$.
(2) $\Leftrightarrow$ (3): Clear.
(2) $\Rightarrow$ (4): Let M be a $\gamma \vee \gamma'$-semidense set in X. Suppose that $f(M)$ is not $\beta \vee \beta'$-dense in Y. Then there exists a proper $\beta \vee \beta'$-closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (2), there exists a proper $\gamma \vee \gamma'$-semiclosed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is $\gamma \vee \gamma'$-semidense in X.
(3) $\Rightarrow$ (2): Suppose (2) is not true. This means that there exists a $\beta \vee \beta'$-closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper $\gamma \vee \gamma'$-semiclosed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is $\gamma \vee \gamma'$-semidense in X. But by (3), $f(f^{-1}(C)) = C$ must be $\beta \vee \beta'$-dense in Y, which is a contradiction to the choice of C. $\square$

Definition 3.5. *A function $f : (X, \tau, \gamma, \gamma') \rightarrow (Y, \sigma, \beta, \beta')$ is said to be somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen provided that if $U \in \tau_{\gamma \vee \gamma'}$ and $U \neq \emptyset$, then there exists a $\beta \vee \beta'$-semiopen set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.*

Definition 3.6. *A function $f : (X, \tau, \gamma, \gamma') \rightarrow (Y, \sigma, \beta, \beta')$ is said to be $\gamma \vee \gamma'$-semiopen provided that if $U \in \tau_{\gamma \vee \gamma'}$, then there exists a $\beta \vee \beta'$-semiopen set V in Y such that $V \subset f(U)$.*

It is clear that every $\gamma \vee \gamma'$-semiopen function is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen but the converse is not true as the following example shows.

Example 3.7. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$;$\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Let $\gamma, \gamma' : \tau \rightarrow \mathcal{P}(X)$ and $\beta, \beta' : \sigma \rightarrow \mathcal{P}(X)$ be operations defined as follows:*

$$A^{\gamma} = \begin{cases} A & \text{if } a \in A, \\ A \cup \{a\} & \text{if } a \notin A, \end{cases} \quad A^{\gamma'} = \begin{cases} A & \text{if } c \in A, \\ A \cup \{c\} & \text{if } c \notin A, \end{cases}$$

$$A^{\beta} = \begin{cases} A & \text{if } A = \{a\}, \\ X & \text{otherwise}, \end{cases} \quad \text{and } A^{\beta'} = A, \quad \text{for all } \ A \in \sigma.$$

Then the identity function $f : (X, \tau, \gamma, \gamma') \to (Y, \sigma, \beta, \beta')$ is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen but not $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen.

Proposition 3.8. *For a bijective function $f : (X, \tau, \gamma, \gamma') \to (Y, \sigma, \beta, \beta')$, the following statements are equivalent:*

1. *f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen.*
2. *If C is a $\gamma \vee \gamma'$-closed subset of X such that $f(C) \neq Y$, then there is a $\beta \vee \beta'$-semiclosed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.*

Proof. (1)⇒(2): Let C be any $\gamma \vee \gamma'$-closed subset of X such that $f(C) \neq Y$. Then $X \backslash C$ is $\gamma \vee \gamma'$-open in X and $X \backslash C \neq \emptyset$. Since f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen, there exists a $\beta \vee \beta'$-semiopen set $V \neq \emptyset$ in Y such that $V \subset f(X \backslash C)$. Put $D = Y \backslash V$. Clearly D is $\beta \vee \beta'$-semiclosed in Y and we claim $D \neq Y$. If $D = Y$, then $V = \emptyset$, which is a contradiction. Since $V \subset f(X \backslash C)$, $D = Y \backslash V \supset (Y \backslash f(X \backslash C)) = f(C)$.
(2)⇒(1): Let U be any nonempty $\gamma \vee \gamma'$-open subset of X. Then $C = X \backslash U$ is a $\gamma \vee \gamma'$-closed set in X and $f(X \backslash U) = f(C) = Y \backslash f(U)$ implies $f(C) \neq Y$. Therefore, by (2), there is a $\beta \vee \beta'$-semiclosed set D of Y such that $D \neq Y$ and $f(C) \subset D$. Clearly $V = Y \backslash D$ is a $\beta \vee \beta'$-semiopen set and $V \neq \emptyset$. Also, $V = Y \backslash D \subset Y \backslash f(C) = Y \backslash f(X \backslash U) = f(U)$. □

Proposition 3.9. *For a function $f : (X, \tau, \gamma, \gamma') \to (Y, \sigma, \beta, \beta')$, the following statements are equivalent:*

1. *f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen.*
2. *If A is a $\beta \vee \beta'$-semidense subset of Y, Then $f^{-1}(A)$ is a $\gamma \vee \gamma'$-dense subset of X.*

Proof. (1)⇒(2): Suppose A is a $\beta \vee \beta'$-semidense set in Y. We want to show that $f^{-1}(A)$ is a $\gamma \vee \gamma'$-dense subset of X. Suppose not, then there exists a $\gamma \vee \gamma'$-closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen and $X \backslash B$ is $\gamma \vee \gamma'$-open, there exists a nonempty $\beta \vee \beta'$-semiopen set C in Y such that $C \subset f(X \backslash B)$. Therefore, $C \subset f(X \backslash B) \subset f(f^{-1}(Y \backslash A)) \subset Y \backslash A$. That is, $A \subset Y \backslash C \subset Y$. Now, $Y \backslash C$ is a $\beta \vee \beta'$-semiclosed set and $A \subset Y \backslash C \subset Y$. This implies that A is not a $\beta \vee \beta'$-semidense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ must be a $\gamma \vee \gamma'$-dense set in X.
(2)⇒(1): Suppose A is a nonempty $\gamma \vee \gamma'$-open subset of X. We want to show that $\beta \vee \beta'$-$s\,\mathrm{Int}(f(A)) \neq \emptyset$. Suppose $\beta \vee \beta'$-$s\,\mathrm{Int}(f(A)) = \emptyset$. Then, $\beta \vee \beta'$-$s\,\mathrm{Cl}(Y \backslash f(A)) = Y$. Therefore, by (2), $f^{-1}(Y \backslash f(A))$ is $\gamma \vee \gamma'$-dense in X. But $f^{-1}(Y \backslash f(A)) \subset X \backslash A$. Now, $X \backslash A$ is $\gamma \vee \gamma'$-closed. Therefore, $f^{-1}(Y \backslash f(A)) \subset X \backslash A$ gives $X = \gamma \vee \gamma'$-$\mathrm{Cl}(f^{-1}(Y \backslash f(A))) \subset X \backslash A$. This implies that $A = \emptyset$, which is contrary to $A \neq \emptyset$. Therefore, $\beta \vee \beta'$-$s\,\mathrm{Int}(f(A)) \neq \emptyset$. This proves that f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen. □

Definition 3.10. *A bioperaation-topological space $(X, \tau, \gamma, \gamma')$ is said to be $\gamma \vee \gamma'$-semi resolvable if there exists a $\gamma \vee \gamma'$-semidense set A in X such that $X \backslash A$ is also $\gamma \vee \gamma'$-semidense in $(X, \tau, \gamma, \gamma')$. Otherwise, $(X, \tau, \gamma, \gamma')$ is called $\gamma \vee \gamma'$-semiirresolvable.*

Theorem 3.11. *For a bioperation-topological space $(X, \tau, \gamma, \gamma')$, the following statements are equivalent:*

1. *$(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semiresolvable;*

2. *$(X, \tau, \gamma, \gamma')$ has a pair of $\gamma \vee \gamma'$-semidense sets A and B such that $A \subset X \backslash B$.*

Proof. (1)⇒(2): Suppose that $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semiresolvable. There exists a $\gamma \vee \gamma'$-semidense set A such that $X \backslash A$ is $\gamma \vee \gamma'$-semidense. Set $B = X \backslash A$, then we have $A = X \backslash B$.
(2)⇒(1): Suppose that the statement (2) holds. Let $(X, \tau, \gamma, \gamma')$ be $\gamma \vee \gamma'$-semiirresolvable. Then $X \backslash B$ is not $\gamma \vee \gamma'$-semidense and $\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(A) \subset \gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(X \backslash B) \neq X$. Hence A is not $\gamma \vee \gamma'$-semidense. This contradicts the assumption. □

Theorem 3.12. *For a bioperation-topological space $(X, \tau, \gamma, \gamma')$, the following statements are equivalent:*

1. *$(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semiirresolvable.*

2. *For any $\gamma \vee \gamma'$-semidense set A in X, $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(A) \neq \emptyset$.*

Proof. (1)⇒(2): Let A be any $\gamma \vee \gamma'$-semidense set of X. Then we have $\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(X \backslash A) \neq X$; hence $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(A) \neq \emptyset$.
(2)⇒(1): Suppose that $(X, \tau, \gamma, \gamma')$ is a $\gamma \vee \gamma'$-semiresolvable space. Then there exists a $\gamma \vee \gamma'$-semidense set A in X such that $X \backslash A$ is also $\gamma \vee \gamma'$-semidense in X. It follows that $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(A) = \emptyset$, which is a contradiction; hence $(X, \tau, \gamma, \gamma')$ is $\gamma \vee \gamma'$-semiirresolvable. □

Theorem 3.13. *If $\overset{n}{\underset{i=1}{\cup}} A_i = X$, where A_i's are subsets of X such that $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(A_i) = \emptyset$, then $(X, \tau, \gamma, \gamma')$ is a $\gamma \vee \gamma'$-semiirresolvable.*

Proof. By hypothesis, we have $\overset{n}{\underset{i=1}{\cap}} (X \backslash A_i) = \emptyset$. Then, there must be at least two nonempty disjoint subsets $X \backslash A_i$ and $X \backslash A_j$ in X. That is $(X \backslash A_i) \cup (X \backslash A_j) \subset X$. Then $X \backslash A_i \subset A_j$; hence $\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(A_j) = X$. Also $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(A_j) = \emptyset$ implies that $\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(X \backslash A_j) = X$. Therefore, $(X, \tau, \gamma, \gamma')$ has a $\gamma \vee \gamma'$-semidense set A_j such that $\gamma \vee \gamma'\text{-}s\,\mathrm{Cl}(X \backslash A_j) = X$. Hence $(X, \tau, \gamma, \gamma')$ is a $\gamma \vee \gamma'$-semiirresolvable. □

Theorem 3.14. *If $f : (X, \tau, \gamma, \gamma') \to (Y, \sigma, \beta, \beta')$ is a somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen function and $\beta \vee \beta'\text{-}\mathrm{Int}(A) = \emptyset$ for a nonempty set A in Y, then $\gamma \vee \gamma'\text{-}\mathrm{Int}(f^{-1}(A)) = \emptyset$.*

Proof. Let A be a nonempty set in Y such that $\beta \vee \beta'\text{-}s\,\mathrm{Int}(A) = \emptyset$. Then $\beta \vee \beta'\text{-}s\,\mathrm{Cl}(Y \backslash A) = Y$. Since f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen and $Y \backslash A$ is $\beta \vee \beta'$-semidense in Y, by Proposition 3.9 $f^{-1}(Y \backslash A)$ is $\gamma \vee \gamma'$-dense in X. Then, $\gamma \vee \gamma'\text{-}\mathrm{Cl}(X \backslash f^{-1}(A)) = X$; hence $\gamma \vee \gamma'\text{-}\mathrm{Int}(f^{-1}(A)) = \emptyset$. □

Theorem 3.15. *Let $f : (X, \tau, \gamma, \gamma') \to (Y, \sigma, \beta, \beta')$ be a somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen function. If X is $\gamma \vee \gamma'$-irresolvable, then Y is $\beta \vee \beta'$-semiirresolvable.*

Proof. Let A be a nonempty set in Y such that $\beta \vee \beta'\text{-}s\,\mathrm{Cl}(A) = Y$. We show that $\beta \vee \beta'\text{-}s\,\mathrm{Int}(A) \neq \emptyset$. Suppose not, then $\beta \vee \beta'\text{-}s\,\mathrm{Cl}(Y \backslash A) = Y$. Since f is somewhat $\gamma \vee \gamma'$-$\beta \vee \beta'$-semiopen and $Y \backslash A$ is $\beta \vee \beta'$-semidense in Y, we have by Proposition 3.9 $f^{-1}(Y \backslash A)$ is $\gamma \vee \gamma'$-dense in X. Then $\gamma \vee \gamma'\text{-}s\,\mathrm{Int}(f^{-1}(A)) = \emptyset$. Now, since A is $\gamma \vee \gamma'$-semidense in Y, $f^{-1}(A)$ is $\gamma \vee \gamma'$-semidense in X. Therefore, for the $\gamma \vee \gamma'$-semidense set $f^{-1}(A)$, we have $\gamma \vee \gamma'\text{-}\mathrm{Int}(f^{-1}(A)) = \emptyset$, which is a contradiction to Theorem 3.12. Hence we must have $\beta \vee \beta'\text{-}s\,\mathrm{Int}(A) \neq \emptyset$ for all $\beta \vee \beta'$-semidense sets A in Y. Hence by Theorem 3.12, Y is $\beta \vee \beta'$-semiirresolvable. □

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