South Asian Journal of Mathematics 2018, Vol. 8 (1): 18 ∼ 30
www.sajm-online.com ISSN 2251-1512 RESEARCH ARTICLE

Some soft sets via soft grills

Alaa Mohamed Abd El-Latif[1][2]*, Rodyna A. Hosny[3]

① Mathematics Department, Faculty of Arts and Science, Northern Border University, Rafha, K. S. A.
② Mathematics Department, Faculty of Education, Ain Shams University, Cairo, Egypt
③ Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt
E-mail: aala.ali@nbu.edu.sa

Received: Dec-5-2017; Accepted: Feb-20-2018 *Corresponding author

Abstract This paper aims to investigate more properties of soft grill which was first introduced in [6]. Also, we introduce and study the concepts of GL-perfect soft sets, GR-perfect soft sets and Gϱ-perfect soft sets in soft topological space which are extensions of the concepts soft τ_G-closed, soft τ_G-dense in itself and soft τ_G-perfect, respectively in terms of soft grill G. Also, a characterization for suitable condition between the soft topology τ and the soft grill G utilizing the collection of GR-perfect soft sets is investigated. Moreover, a new generalized finite soft topology on a finite soft set via the collection GR-perfect soft sets, which is finer than soft topology τ_G, is obtained. Furthermore, we will show for any grill G the collection GL-$\mathcal{PSS}$(X, A) is a finite generalized soft topology.

Key Words Soft sets, Soft topological space, Soft grill, Perfect soft sets
MSC 2010 54A05, 11B05

1 Introduction

Soft set theory was first introduced by Molodtsov [18] as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. Modern topology depends strongly on the ideas of set theory, but M. shabir and M. Naz [20] used soft sets to define a topology namely soft topology and defined soft separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In addition, Maji et al. [16] introduced several operations on soft sets and basic properties of these operations have been revealed thus far. The notion of soft ideal was initiated for the first time by Kandil et al. [11]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. Applications to various fields were further investigated by Kandil et al.[7, 9, 10, 12, 13, 14, 15]. The notions of soft grill, soft operators $\phi_G(\ )$, $\psi_G(\ )$ and soft topology τ_G were defined and discussed in [6].

Citation: Alaa Mohamed Abd El-Latif, Rodyna A. Hosny, Some soft sets via soft grills, South Asian J Math, 2018, 8(1), 18-30.

The main purpose of this paper is to introduce and investigate GL-perfect soft, GR-perfect soft and Gϱ-perfect soft sets in soft topological spaces which described by soft grill. A characterization for a suitable soft grill via GR-perfect soft sets is studied. Also, a new generalized soft topology via the collection of GR-perfect soft sets, which is finer than the soft topology τ_G on a finite universe set X is obtained. Furthermore, we show that for any grill G the collection GL-$\mathcal{PSS}$(X, A) is a finite generalized soft topology.

2 Preliminaries

Let X be an initial universe set and E be a set of parameters. Let $\mathcal{P}$(X) be the power set of X and A be a non empty subset of E. F_A is called a soft set over X [18], where F is a mapping given by F: A$\longrightarrow\mathcal{P}$(X). For two soft sets F_A, H_B over common universe X, we say that F_A is a subset of H_B [16], if A$\subseteq$B and F(e)$\subseteq$H(e), for all e$\in$A. In this case, we write $F_A \sqsubseteq H_B$. Two soft sets F_A, H_B over a common universe X are said to be equal, if $F_A \sqsubseteq H_B$ and $H_B \sqsubseteq F_A$.
A soft set F_A over X is called a null (resp., an absolute) soft set, denoted by $\tilde{\emptyset_A}$ (resp., $\tilde{X_A}$), if for each e$\in$A, F(e)=$\emptyset$ (resp., F(e)=X) [16]. In particular,
(X, A) will be denoted by $\tilde{X_A}$.

The union of two soft sets of F_A and H_B over the common universe X [16] is the soft set K_C, where C=A$\cup$B and for all e$\in$C, K(e)=F(e), if e$\in$(A - B); K(e)=H(e), if e$\in$(B - A), and K(e)=F(e)$\cup$H(e), if e$\in$(A$\cap$B). We write $F_A \sqcup H_B = K_C$. Moreover, the intersection K_C of two soft sets F_A and H_B over a common universe X [20], denoted $F_A \sqcap H_B$, is defined as C=A$\cap$B, and K(e)=F(e)$\cap$H(e) for all e$\in$C. The difference between two soft sets F_A and H_A over X [20], denoted by K_A=(F_A - H_A), is defined as K(e)=F(e) - H(e) for all e$\in$A. The symmetric difference K_A of two soft sets F_A and H_A, denoted by $F_A \Delta H_A$, is defined as (F_A - H_A)$\sqcup$(H_A - F_A). The relative complement of a soft set F_A [1], is denoted by $(F_A)^c$ and is defined by $(F_A)^c = \tilde{X_A}$ - F_A, where F^c: A$\longrightarrow\mathcal{P}$(X) is a mapping given by $(F)^c$(e)= X - F(e) for all e$\in$A. Moreover, $((F_A)^c)^c = F_A$.

Remark In order to efficiently discuss, we consider only soft sets F_A over a universe X in which all the parameters set A are the same.

In this paper for convenience, let $\mathcal{SS}$(X, A) be the family of soft sets over X with set of parameters A.

Definition 2.1 [20] Let F_A be a soft set over X and x$\in$X. x$\in F_A$ whenever x$\in$F(e) for all e$\in$A. Note that for any x$\in$X, x$\notin F_A$, if x$\notin$F(e) for some e$\in$A.

Definition 2.2 [20] Let x$\in$X, then x_A denotes the soft set over X for which x(e)={x} for all e$\in$A.

Definition 2.3 [20] Let τ be the collection of soft sets over X with the fixed set of parameters A.

Then, τ is said to be a soft topology on X, if it satisfies the following axioms:
(i) $\tilde{\emptyset}_A$, $\tilde{X}_A$ belong to τ.
(ii) the union of any number of soft sets in τ belongs to τ.
(iii) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, A, τ) is called a soft topological space over X or a soft space. Every member of τ is called an open soft set. The complement of an open soft set is called the closed soft set [8] in (X, A, τ).

Definition 2.4 [21] Let (X, τ, A) be a soft topological space. A sub-collection $\beta \sqsubseteq \tau$ is said to be a soft base for τ, if every open soft set can be expressed as a soft union of members of β.

Definition 2.5 [21] A soft set F_A of a soft topological space (X, τ, A) is called a soft neighborhood (soft nbd.) of the soft point x_e if there exists an open soft set K_A such that $x_e \in K_A \sqsubseteq F_A$. The neighborhood system of a soft point x_e, denoted by $\mathcal{N}_{xe}$, is the family of all its neighborhoods.

Definition 2.6 [20] The soft closure cl F_A of a soft set F_A in a soft topological space (X, τ, A), is the intersection of all soft closed sets that contain F_A, i.e., cl $F_A = \sqcap\{M_A \in \tau^c \mid F_A \sqsubseteq M_A\}$. Clearly, a soft point x_e is called soft closure point of F_A, if for every open soft nbd. U_A of x_e; $F_A \sqcap U_A \neq \tilde{\emptyset}_A$.

Definition 2.7[11] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if
(1) $(F, E) \in \tilde{I}$ and $(G, E) \in \tilde{I} \Rightarrow (F, E)\tilde{\cup}(G, E) \in \tilde{I}$,
(2) $(F, E) \in \tilde{I}$ and $(G, E)\tilde{\subseteq}(F, E) \Rightarrow (G, E) \in \tilde{I}$,
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Definition 2.8 [6] A non-empty collection $G \sqsubseteq \mathcal{SS}$(X, A) of soft sets over X is called a soft grill, if the following conditions hold:
(i) If $F_A \in G$ and $F_A \sqsubseteq H_A$, then $H_A \in G$.
(ii) If $F_A \sqcup H_A \in G$, then $F_A \in G$ or $H_A \in G$.

Definition 2.9 [6] Let G be a soft grill over a soft topological space (X, τ, A). Consider the soft operator ϕ_G: $\mathcal{SS}$(X, A)$\longrightarrow\mathcal{SS}$(X, A), given by $\phi_G(F_A) = \{x_e \mid U_A \sqcap F_A \in G$ for every open soft nbd. U_A of $x_e\}$ for every soft set F_A. Then, the soft operator ψ_G: $\mathcal{SS}$(X, A)$\longrightarrow\mathcal{SS}$(X, A), defined by for every soft set F_A, $\psi_G(F_A) = F_A \sqcup \phi_G(F_A)$ is a kuratowski s closure operator and hence gives rise to a new soft topology $\tau_G = \{H_A \mid \psi_G(\tilde{X}_A - H_A) = (\tilde{X}_A - H_A)\}$ on (X, A) which is finer than τ in general. A soft base β(G, τ) for the soft topology τ_G on (X, A) is given by $\beta(G, \tau) = \{(V_A - F_A) \mid V_A \in \tau, F_A \notin G\}$ and $\tau \sqsubseteq \beta(G, \tau) \sqsubseteq \tau_G$.

Lemma 2.1 [6] Let G be a soft grill over a soft topological space (X, τ, A). Then, for every $F_A \in \mathcal{SS}$(X, A) the following statements hold:
(i) If $F_A \notin G$, then $\phi_G(F_A) = \tilde{\emptyset}_A$. Moreover, $\phi_G(\tilde{\emptyset}_A) = \tilde{\emptyset}_A$.
(ii) $\phi_G \phi_G(F_A) \sqsubseteq \phi_G(F_A) =$ cl $\phi_G(F_A) \sqsubseteq$ cl F_A. Moreover, $\phi_G(F_A)$ is soft τ-closed.

(iii)$\phi_G(\psi_G(F_A))=\psi_G(\phi_G(F_A))=\phi_G(F_A)$.
(iv) $\psi_G(\psi_G(F_A))=\psi_G(F_A)$.
(v) If F_A is a soft closed set, then $\phi_G(F_A)\sqsubseteq F_A$.
(vi) F_A is a soft τ_G-closed set if and only if $\phi_G(F_A)\sqsubseteq F_A$.

Lemma 2.2 [6] Let G be a soft grill over a soft topological space (X, τ, A). Then, for every F_A, $H_A \in \mathcal{SS}$(X, A) the following statements hold:
(i) $F_A\sqsubseteq H_A$ implies $\phi_G(F_A)\sqsubseteq\phi_G(H_A)$.
(ii) $\phi_G(F_A\sqcup H_A)=\phi_G(F_A)\sqcup\phi_G(H_A)$ and $\phi_G(F_A\sqcap H_A)\sqsubseteq\phi_G(F_A)\sqcap\phi_G(H_A)$.
(iii) $\phi_G(F_A)$ - $\phi_G(H_A)=\phi_G(F_A$ - $H_A)$ - $\phi_G(H_A)$.
(iv) If $H_A\notin$G, then $\phi_G(F_A\sqcup H_A)=\phi_G(F_A)=\phi_G(F_A$ - $H_A)$.
(v) If $F_A\Delta H_A\notin$G, then $\phi_G F_A=\phi_G H_A$.

Lemma 2.3 [6] Let (X, τ, A) be a soft topological space, $F_A\in\mathcal{SS}$(X, A) and G_1, G_2 be two soft grills over X. Then, $\phi_{G1}(F_A)\sqsubseteq\phi_{G2}(F_A)$, if $G_1\sqsubseteq G_2$.

Lemma 2.4[6] If G is a soft grill on a soft topological space (X, τ, A) with (τ - $\tilde{\emptyset_A}$)$\sqsubseteq$G. Then,
(i) For any open soft set H_A, $H_A\sqsubseteq\phi_G(H_A)$.
(ii) $\phi_G(\tilde{X_A})=\tilde{X_A}$.

Theorem 2.1[6] Let G be a soft grill on a soft topological space (X, τ, A) and $F_A\in\mathcal{SS}$(X, A) such that $F_A\sqsubseteq\phi_G(F_A)$. Then, cl $F_A=\tau_G$-cl F_A=cl($\phi_G(F_A)$)= $\phi_G(F_A)$.

3 On soft grill

The notion of soft grill was introduced by R. Hosny in [6]. In this section, we will formulate, in different ways, a suitable condition between the given soft topology τ and the soft grill G, such this condition makes the induced soft topology τ_G well-behaved and more applicable. We intend to do some investigations in respect of the soft topology τ_G, along with certain applications, under the assumption of such a suitability condition imposed on the concerned soft grills.

It is worth to mention that, in the current paper the following formula will be used, for every $F_A\in\mathcal{SS}$(X, A), $\widetilde{F_A}$=(F_A - $\varphi_G(F_A)$) and $\widehat{F_A}$=($\varphi_G(F_A)$ - F_A).

Definition 3.1 Let G be a soft grill over a soft topological space (X, τ, A). Then, a soft topology τ is said to be suitable for the soft grill G, if for all $F_A\in\mathcal{SS}$(X, A), (F_A - $\varphi_G(F_A)$)$\notin$G.

Some equivalent descriptions of the suitability between soft topological space and soft grill will show in the next theorem.

Theorem 3.1 Let G be a soft grill over a soft topological space (X, τ, A), then the following statements are equivalent

(i) A soft topology τ is suitable for the soft grill G.

(ii) For any τ_G-closed soft set F_A, $(F_A - \varphi_G(F_A)) \notin G$.

(iii) For any $F_A \in \mathcal{SS}(X, A)$ and each soft point x_α, $x_\alpha \in F_A$ there exists open soft nbd. U_A of x_α with $U_A \sqcap F_A \notin G$, it follows that $F_A \notin G$.

(iv) $F_A \in \mathcal{SS}(X, A)$ and $F_A \sqcap \varphi_G(F_A) = \tilde{\emptyset}_A$, then $F_A \notin G$.

Proof (i)$\Rightarrow$(ii) Obvious. (ii)$\Rightarrow$(iii) Let $F_A \in \mathcal{SS}(X, A)$ and assume that for every $x_\alpha \in F_A$ there exists open soft nbd. U_A of x_α such that $U_A \sqcap F_A \notin G$. Then, $x_\alpha \notin \varphi_G(F_A)$ so that $F_A \sqcap \varphi_G(F_A) = \tilde{\emptyset}_A$. Now as $\Psi_G(F_A)$ is τ_G-closed soft set, then by (ii) of this theorem $(\Psi_G(F_A) - \varphi_G(\Psi_G(F_A))) \notin G$. By using Lemmas 2.1, 2.2, we have $F_A = (F_A \sqcup \varphi_G(F_A) - \varphi_G(F_A)) = [(F_A \sqcup \varphi_G(F_A)) - \varphi_G(F_A \sqcup \varphi_G(F_A))] = (\Psi_G(F_A) - \varphi_G(\Psi_G(F_A))) \notin G$. Consequently, $F_A \notin G$.

(iii)$\Rightarrow$(iv) Let $F_A \in \mathcal{SS}(X, A)$ and $F_A \sqcap \varphi_G(F_A) = \tilde{\emptyset}_A$, then $x_\alpha \notin \varphi_G(F_A)$ for every $x_\alpha \in F_A$. Hence, there exists open soft nbd. U_A of x_α such that $U_A \sqcap F_A \notin G$. In view of (iii), $F_A \notin G$.

(iv)$\Rightarrow$(i) Let $F_A \in \mathcal{SS}(X, A)$, then by using Lemma 2.1 and Lemma 2.2, $(F_A - \varphi_G(F_A)) \sqcap \varphi_G(F_A - \varphi_G(F_A)) = (F_A - \varphi_G(F_A)) \sqcap \varphi_G(F_A - \varphi_G(F_A)) = \tilde{\emptyset}_A$. Hence by (iv), $(F_A - \varphi_G(F_A)) \notin G$.

Theorem 3.2 Let G be a soft grill over a soft topological space (X, A, τ) and $F_A \in \mathcal{SS}(X, A)$. Then, the following statements are equivalent

(i) $F_A \sqcap \phi_G(F_A) = \tilde{\emptyset}_A$, then $\phi_G(F_A) = \tilde{\emptyset}_A$.

(ii) $\phi_G(F_A - \phi_G(F_A)) = \tilde{\emptyset}_A$.

(iii) $\phi_G(F_A \sqcap \phi_G(F_A)) = \phi_G(F_A)$.

Proof (i)$\Rightarrow$(ii) Let $F_A \in \mathcal{SS}(X, A)$, then $(F_A - \phi_G(F_A)) \sqcap \phi_G(F_A - \phi_G(F_A)) = \tilde{\emptyset}_A$ and so $\phi_G(F_A - \phi_G(F_A)) = \tilde{\emptyset}_A$.

(ii)$\Rightarrow$(iii) Let $F_A \in \mathcal{SS}(X, A)$ and $F_A = (F_A - (F_A \sqcap \phi_G(F_A))) \sqcup (F_A \sqcap \phi_G(F_A))$, then $\phi_G(F_A) = \phi_G(F_A - (F_A \sqcap \phi_G(F_A))) \sqcup \phi_G(F_A \sqcap \phi_G(F_A)) = \phi_G(F_A - \phi_G(F_A)) \sqcup \phi_G(F_A \sqcap \phi_G(F_A)) = \phi_G(F_A \sqcap \phi_G(F_A))$.

(iii)$\Rightarrow$(i) Let $F_A \in \mathcal{SS}(X, A)$ and $F_A \sqcap \phi_G(F_A) = \tilde{\emptyset}_A$, then $\phi_G(F_A) = \phi_G(F_A \sqcap \phi_G(F_A)) = \phi_G(\tilde{\emptyset}_A) = \tilde{\emptyset}_A$.

Theorem 3.2 gives us a certain necessary conditions for a suitability a soft topology τ with the soft grill G.

Theorem 3.3 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G and F_A is a τ_G-closed soft set, then $\phi_G \phi_G(F_A) = \phi_G(F_A)$.

Proof Let F_A be a τ_G-closed soft set and a soft topology τ is a suitable for the soft grill G, then by Lemma 2.1 and Theorem 3.1, $\phi_G(F_A) \sqsubseteq F_A$ and $(F_A - \phi_G(F_A)) \notin G$. Hence, $(\phi_G(F_A) - F_A) = \tilde{\emptyset}_A \notin G$ and $(F_A - \phi_G(F_A)) \notin G$. Consequently, $(\phi_G(F_A) - F_A) \sqcup (F_A - \phi_G(F_A)) \notin G$ and so $F_A \Delta \phi_G(F_A) \notin G$. Thus, by Lemma 2.2 (v) $\phi_G \phi_G(F_A) = \phi_G(F_A)$.

Theorem 3.4 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G and $F_A \in \mathcal{SS}$(X, A). Then, $\phi_G\phi_G(F_A)=\phi_G(F_A)$.

Proof Let $F_A=(F_A \sqcap \phi_G(F_A)) \sqcup (F_A - \phi_G(F_A))$, then $\phi_G(F_A)=\phi_G(F_A \sqcap \phi_G(F_A)) \sqcup \phi_G(F_A - \phi_G(F_A))$. Since a soft topology τ is a suitable for the soft grill G, then $(F_A - \phi_G(F_A)) \notin G$. In view of Lemma 2.1 $\phi_G(F_A - \phi_G(F_A))=\tilde{\emptyset}_A$. Hence, $\phi_G(F_A)=\phi_G(F_A \sqcap \phi_G(F_A)) \sqsubseteq \phi_G\phi_G(F_A)$. Since $\phi_G\phi_G(F_A) \sqsubseteq \phi_G(F_A)$, consequently, $\phi_G\phi_G(F_A)=\phi_G(F_A)$.

Corollary 3.1 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G and $F_A \in \mathcal{SS}$(X, A). Then, $\phi_G(F_A)$ is a soft τ_G-perfect.

Theorem 3.5 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G. Then, a soft set F_A is soft τ_G-closed if and only if it can be expressed as a union of a soft τ-closed and a soft set which is not in G.

Proof Let F_A be a τ_G-closed soft set, then by using Lemma 2.1, $\phi_G(F_A) \sqsubseteq F_A$ and so $F_A=\phi_G(F_A) \sqcup (F_A - \phi_G(F_A))$. Since a soft topology τ is suitable for soft grill G, then $(F_A - \phi_G(F_A)) \notin G$. In view of Lemma 2.1, $\phi_G(F_A)$ is soft τ_G-closed. Conversely, let $F_A=U_A \sqcup H_A$, where U_A is soft τ-closed and $H_A \notin G$. Then, by using Lemmas 2.1, 2.2, we have $\phi_G(F_A)=\phi_G(U_A \sqcup H_A)=\phi_G(U_A) \sqsubseteq \mathrm{cl}\ U_A=U_A \sqsubseteq F_A$. Hence, F_A is soft τ_G-closed.

Corollary 3.2 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G. Then, a soft set F_A is soft τ_G-closed if and only if it can be expressed as a union of a soft τ_G-perfect set and a soft set which is not in G.

Theorem 3.6 Let G be a soft grill over a soft topological space (X, τ, A) such that a soft topology τ is a suitable for the soft grill G. Then, a soft open base β(G, τ) for the soft topology τ_G on (X, A) is τ_G.

Proof Let F_A be a τ_G-open soft set. Then, by Theorem 3.5, $(\tilde{X}_A - F_A)=U_A \sqcup H_A$, where U_A is a τ-closed soft set and $H_A \notin G$. Then, $F_A=(\tilde{X}_A - U_A) \sqcap (\tilde{X}_A - H_A)=(\tilde{X}_A - U_A) - H_A=V_A - H_A$, where $V_A=(\tilde{X}_A - U_A)$ is a τ-open soft set. Thus every τ_G-open soft set is of the form $(V_A - H_A)$. Hence, $F_A \in \beta$(G, τ). Since β(G, τ) is a open soft base for the soft topology τ_G on (X, A), then β(G, τ)=τ_G.

In view of Lemma 2.4 and Theorem 2.1, it follows that whenever each non-empty open soft set is a member of the soft grill G, then the condition and hence the conclusions of Theorem 2.1 hold. If in addition, the soft topology is suitable for the soft grill G, we have something more as is given by the following theorem.

Theorem 3.7 Let G be a soft grill over a soft topological space (X, τ, A) with $(\tau - \tilde{\emptyset}_A) \sqsubseteq G$ such that a soft topology τ be a suitable for the soft grill G. If H_A is a τ_G-open soft set and H_A=(U_A - F_A), where U_A is a τ-open soft set and $F_A \notin$G, then τ_G-cl H_A=cl H_A=$\phi_G(H_A)$=$\phi_G(U_A)$=cl U_A=τ_G-cl U_A.

Proof Let H_A be a τ_G-open soft set and H_A=(U_A - F_A), where U_A is a τ-open soft set and $F_A \notin$G, then in view of Lemma 2.2 we have $\phi_G(H_A)$=$\phi_G(U_A)$. Since (τ - $\tilde{\emptyset_A}$)$\sqsubseteq$G and U_A is a τ-open soft set, then using by Lemma 2.13 we have $U_A \sqsubseteq \phi_G(U_A)$ and so by using Theorem 2.14, $\phi_G(U_A)$=cl U_A=τ_G-cl U_A. Since H_A is a τ_G-open soft set, then ($\tilde{X_A}$ - H_A) is a τ_G-closed soft set and so by using Lemma 2.1, $\phi_G(\tilde{X_A}$ - $H_A)\sqsubseteq(\tilde{X_A}$ - $H_A)$. Therefore, $\phi_G(\tilde{X_A})$ - $\phi_G(H_A)\sqsubseteq$ ($\tilde{X_A}$ - H_A). Since (τ - $\tilde{\emptyset_A}$)$\sqsubseteq$G, then by Lemma 2.5, $\phi_G(\tilde{X_A})=\tilde{X_A}$ and so $\tilde{X_A}$ - $\phi_G(H_A)\sqsubseteq(\tilde{X_A}$ - $H_A)$ which implies that $H_A \sqsubseteq \phi_G(H_A)$. Hence, by Theorem 2.1, $\phi_G(H_A)$=cl H_A=τ_G-cl H_A. Consequently, τ_G-cl H_A=cl H_A=$\phi_G(H_A)$=$\phi_G(U_A)$=cl U_A=τ_G-cl U_A.

Theorem 3.8 For any soft grill G on a soft topological space (X, τ, A), let $(F_A)^{'G}$ and $(F_A)^{'}$ denote the derived sets of A with respect to τ_G and τ respectively. Then,
(i) $(F_A)^{'G}\sqsubseteq(F_A)^{'}$.
(ii) $(F_A)^{'G}\sqsubseteq\phi_G(F_A)$.

Proof (i) Follows from the fact that $\tau\sqsubseteq\tau_G$. (ii) Let $x_\alpha\in(F_A)^{'G}$. Hence, $x_\alpha\in\Psi_G(F_A$ - $\{x_\alpha\})$=(F_A - $\{x\})\sqcup\phi_G(F_A$ - $\{x_\alpha\})$. Consequently, $x_\alpha\in\phi_G(F_A$ - $\{x_\alpha\})\sqsubseteq\phi_G(F_A)$.

Definition 3.2 Let G be a soft grill over a soft topological space (X, τ, A). Then, a soft topology τ is said to be suitable for a soft G, if (F_A - $\phi_G(F_A))\notin$G, for all $F_A\in S\mathcal{SS}$(X, A).

Some equivalent descriptions of the suitability of soft topology with a soft grill will show in the next Theorem.

Theorem 3.9 Let G be a soft grill over a soft topological space (X, τ, A), then the following statements are equivalent
(i) A soft topology τ is suitable for a soft G.
(ii) For any soft τ_G-closed set F_A, (F_A - $\phi_G(F_A))\notin$G.
(iii) For any soft set F_A and each $x_e\in F_A$ there exists an open soft nbd. U_A of x_e with $U_A\sqcap F_A\notin$G, it follows that $F_A\notin$G.
(iv) If F_A is a soft set and $F_A\sqcap\phi_G(F_A)=\tilde{\emptyset_A}$, then $F_A\notin$G.

Proof: (i)$\Rightarrow$(ii) Obvious. (ii)$\Rightarrow$(iii) Let $F_A\in\mathcal{SS}$(X, A) and assume that for every $x_e\in F_A$ there exists open soft nbd. U_A of x_e such that $U_A\sqcap F_A\notin\mathcal{G}$. Then, $x_e\notin\varphi_{\mathcal{G}}(F_A)$ so that $F_A\sqcap\varphi_{\mathcal{G}}(F_A)=\tilde{\emptyset_A}$. Now as $\Psi_{\mathcal{G}}(F_A)$ is $\tau_{\mathcal{G}}$-soft closed, then by (ii) of this theorem ($\Psi_{\mathcal{G}}(F_A)$ - $\varphi_{\mathcal{G}}(\Psi_{\mathcal{G}}(F_A)))\notin\mathcal{G}$. By using Lemma 2.1 and Lemma 2.2, we have F_A=($F_A\sqcup\varphi_{\mathcal{G}}(F_A)$ - $\varphi_{\mathcal{G}}(F_A)$)=[($F_A\sqcup\varphi_{\mathcal{G}}(F_A)$) - $\varphi_{\mathcal{G}}(F_A\sqcup\varphi_{\mathcal{G}}(F_A))$]= ($\Psi_{\mathcal{G}}(F_A)$ - $\varphi_{\mathcal{G}}(\Psi_{\mathcal{G}}(F_A)))\notin\mathcal{G}$. Consequently, $F_A\notin\mathcal{G}$.
(iii)$\Rightarrow$(iv) Let $F_A\in\mathcal{SS}$(X, A) and $F_A\sqcap\varphi_{\mathcal{G}}(F_A)=\tilde{\emptyset_A}$, then $x_e\notin\varphi_{\mathcal{G}}(F_A)$ for every $x_e\in F_A$. Hence, there exists open soft nbd. U_A of x_e such that $U_A\sqcap F_A\notin\mathcal{G}$. In view of (iii), $F_A\notin\mathcal{G}$.

(iv)$\Rightarrow$(i) Let $F_A \in \mathcal{SS}(X, A)$, then by using Lemma 2.1 and Lemma 2.2, $(F_A - \varphi_{\mathcal{G}}(F_A)) \sqcap \varphi_{\mathcal{G}}(F_A - \varphi_{\mathcal{G}}(F_A)) = \tilde{\emptyset}_A$. Hence by (iv), $(F_A - \varphi_{\mathcal{G}}(F_A)) \notin \mathcal{G}$.

4 GL, GR and Gϱ-perfect soft sets

Definition 4.1 Let G be a soft grill over a soft topological space (X, τ, A). Then, a soft set F_A is said to be:
(i) Soft G-dense in itself, if $F_A \sqsubseteq \phi_G(F_A)$.
(ii) Soft G-dense, if $\phi_G(F_A) = \tilde{X_A}$.
(iii) Soft G-perfect, if $\phi_G(F_A) = F_A$.
(iv) GL-perfect soft set, if $(F_A - \phi_G(F_A)) \notin G$.
(v) GR-perfect soft set, if $(\phi_G(F_A) - F_A) \notin G$.
(vi) Gϱ-perfect soft, if $F_A \Delta \phi_G(F_A) \notin G$.

The collection of GL (resp., GR and Gϱ)-perfect soft sets in (X, τ, A) are denoted by GL-$\mathcal{PSS}$(X, A) (resp., GR-$\mathcal{PSS}$(X, A) and Gϱ-$\mathcal{PSS}$(X, A)).

Lemma 4.1 Let G be a soft grill on a soft topological space (X, τ, A). Then, the following statements hold:
(i) Every soft G-dense set is soft G-dense in itself.
(ii) Every soft G-perfect set is soft G-dense in itself.

Proof: It is obvious.

The converse of above Lemma need not be true in general. The following example supports our claim.

Example 4.1 Let X={x, y}, A={e_1, e_2}, G={E_{1A}, E_{2A}, E_{3A}, $\tilde{X_A}$} and τ={F_{1A}, F_{2A}, F_{3A}, $\tilde{\emptyset}_A$, $\tilde{X_A}$} where E_{1A}, E_{2A}, E_{3A}, F_{1A}, F_{2A} and F_{3A} are soft sets over X, defined by E_{1A}={{x}, {y}}, E_{2A}={{x}, X}, E_{3A}={X, {y}}, $F_{1A}=E_{2A}$, F_{2A}={$\emptyset$, X} and F_{3A}={{y}, X}. Then τ is a soft topology over X and G is a soft grill over X. A soft set {{x}, {y}} is soft G-dense in itself and it is not soft G-dense nor soft G-perfect, where $\phi_G(F_A)$={{x}, $\emptyset$}.

Lemma 4.2 Let G be a soft grill on a soft topological space (X, τ, A). Then, the following statements hold:
(i) Every soft G-dense in itself is GL-perfect soft set.
(ii) Every soft τ_G-closed set is GR-perfect soft set.
(iii) Every soft τ-closed set is GR-perfect soft set.
(iv) Every soft set is Gϱ-perfect if and only if it is both GL-perfect and GR-perfect.
(v) Every soft set is both GL-perfect soft set and GR-perfect soft set, if it is soft G-perfect.

(vi) For each soft set (F, A), where $F_A \notin G$, we obtain F_A is GL-perfect soft set and Gϱ-perfect soft set. Moreover, every subset of F_A is also Gϱ-perfect soft set.

Proof Follows directly, in view of Definition 2.7, Definition 3.1 and $\tau \sqsubseteq \tau_G$.

Remark 4.1 The converse of (i), (ii) and (iii) of Lemma 4.2 are not true in general, as shown in the following example.

Examples 4.1 Let X={x, y}, A={e_1, e_2}, G={E_{1A}, E_{2A}, E_{3A}, $\tilde{X_A}$} and τ={F_A, $\tilde{\emptyset_A}$, $\tilde{X_A}$} where E_{1A}, E_{2A}, E_{3A} and F_A are soft sets over X, defined by E_{1A}={{y}, X}, E_{2A}={{x}, X}, E_{3A}={X, {y}} and F_A=E_{1A}. Then, τ is a soft topology over X and G is a soft grill over X.
(i) A soft set {X, {y}} is GL-perfect and it is not soft G-dense in itself, where $\phi_G(F_A)$={{x}, $\emptyset$}.
(ii) A soft set {{y}, X} is GR-perfect and it is not soft τ_G-closed, where $\phi_G(F_A)$={X, X}.

Lemma 4.3 Let (X, τ, A) be a soft topological space with G=$\mathcal{P}$(X) - {$\tilde{\emptyset_A}$} and $F_A \in \mathcal{SS}$(X, A). Then,
(i) F_A is soft G-perfect set.
(ii) GL-$\mathcal{PSS}$(X, A)=GR-$\mathcal{PSS}$(X, A)=Gϱ-$\mathcal{PSS}$(X, A)).

Proof: (i) Since G=$\mathcal{P}$(X) - {$\tilde{\emptyset_A}$}, then $\phi_G(F_A)=F_A$ and so F_A is soft G-perfect set.
(ii) It is clear.

Definition 4.2 A soft principal grill generated by a soft set F_A in a soft topological space (X, τ, A) is defined as [F_A]={U_A | $F_A \sqcap U_A \neq \tilde{\emptyset_A}$}.

Lemma 4.4 Let (X, τ, A) be a soft topological space and $F_A \in \mathcal{SS}$(X, A). If G=[F_A], then $\phi_G(F_A)=\mathrm{cl}_\tau F_A$ and GR-$\mathcal{PSS}$(X, A)=Gϱ-$\mathcal{PSS}$(X, A))=$\mathcal{SS}$(X, A).

Proof: Obvious from the Definition 4.5.

Lemma 4.5 Let G be a soft grill on a soft topological space (X, τ, A). Then, the following statements hold:
(i) $\tilde{\emptyset_A}$ is GL-perfect soft, GR-perfect soft and Gϱ-perfect soft sets.
(ii) $\tilde{X_A}$ is GR-perfect soft set.
(iii) $\tilde{X_A}$ is GL-perfect soft set, if $\phi_G(\tilde{X_A})=\tilde{X_A}$.
(iv) $\phi_G(F_A)$, $\psi_G(F_A)$ and cl (F_A) are GR-perfect soft set.

Proof: Obvious by Lemma 2.1 and Lemma 4.3.

Theorem 4.1 Let G be a soft grill on a soft topological space (X, τ, A) and a soft set $F_A \notin$G. Then,
(i) (F_A - $\phi_G(F_A)$) is Gϱ-perfect soft set.

(ii) (ϕ_G(F$_A$) - F$_A$) is Gϱ-perfect soft set.
(iii) (F$_A\Delta\phi_G$(F$_A$)) is Gϱ-perfect soft set.

Proof: Obvious.

Theorem 4.2 Let (X, τ, A) be a soft topological space, F$_A\in\mathcal{SS}$(X, A) and G$_1$, G$_2$ be two soft grills over X with G$_1\sqsubseteq$G$_2$. Then, F$_A$ is GR-perfect soft set with respect to G$_1$, if it is GR-perfect soft set with respect to G$_2$.

Proof: Since G$_1\sqsubseteq$G$_2$ and by using Lemma 2.3, then ϕ_{G1}(F$_A$)$\sqsubseteq\phi_{G2}$(F$_A$). Let F$_A$ be GR-perfect soft set with respect to G$_2$, then (ϕ_{G2}(F$_A$) - F$_A$)$\notin$G$_2$. Since, (ϕ_{G1}(F$_A$) - F$_A$)$\sqsubseteq$(ϕ_{G2}(F$_A$) - F$_A$), hence (ϕ_{G1}(F$_A$) - F$_A$)$\notin$G$_2$. Since G$_1\sqsubseteq$G$_2$, thus (ϕ_{G1}(F$_A$) - F$_A$)$\notin$G$_1$. Consequently, F$_A$ is GR-perfect soft set with respect to G$_1$.

Lemma 4.6 Let G be a soft grill on a soft topological space (X, τ, A) and F$_A$, H$_A$ be soft sets such that F$_A\sqsubseteq$H$_A\sqsubseteq\phi_G$(F$_A$), then ϕ_G(F$_A$)=ϕ_G(H$_A$).

Proof: It is clear from Lemma 2.1 and Lemma 2.2.

Theorem 4.3 Let G be a soft grill on a soft topological space (X, τ, A) and F$_A$, H$_A$ be soft sets such that F$_A\sqsubseteq$H$_A\sqsubseteq\phi_G$(F$_A$). Then, the following statements hold:
(i) If H$_A$ is GL-perfect soft set, then F$_A$ is GL-perfect soft set.
(ii) If F$_A$ is GR-perfect soft set, then H$_A$ is GR-perfect soft set.

Proof: (i) Let H$_A$ be GL-perfect soft set, then (H$_A$ - ϕ_G(H$_A$))$\notin$G. Since, F$_A\sqsubseteq$H$_A$, then (F$_A$ - ϕ_G(H$_A$))$\sqsubseteq$(H$_A$ - ϕ_G(H$_A$))$\notin$G. Hence, by using Lemma 4.10, ϕ_G(F$_A$)=ϕ_G(H$_A$), and so (F$_A$ - ϕ_G(F$_A$))$\notin$G. Consequently, F$_A$ is GL-perfect soft set.

(ii) Let F$_A$ be GR-perfect soft set, then (ϕ_G(F$_A$) - F$_A$)$\notin$G. Since F$_A\sqsubseteq$H$_A$, then (ϕ_G(F$_A$) - H$_A$)$\sqsubseteq$(ϕ_G(F$_A$) - F$_A$)$\notin$G. Hence by using Lemma 4.10 ϕ_G(F$_A$)=ϕ_G(H$_A$) and so (ϕ_G(H$_A$) - H$_A$)$\notin$G. Consequently, H$_A$ is GR-perfect soft set.

Theorem 4.4 Let G be a soft grill on a soft topological space (X, τ, A) and F$_A$, H$_A\in\mathcal{SS}$(X, A). Then,
(i) If F$_A$, H$_A$ are GL-perfect soft set, then F$_A\sqcup$H$_A$ is GL-perfect soft set.
(ii) If F$_A$, H$_A$ are GR-perfect soft set, then F$_A\sqcup$H$_A$ is GR-perfect soft set.

Proof: (i) Let F$_A$, H$_A$ be GL-perfect soft sets. Then, (F$_A$ - ϕ_G(F$_A$))$\notin$G and (H$_A$ - ϕ_G(H$_A$))$\notin$G. By the definition of soft grill, (F$_A$ - ϕ_G(F$_A$))$\sqcup$(H$_A$ - ϕ_G(H$_A$))$\notin$G. Since, (F$_A\sqcup$H$_A$ - ϕ_G(F$_A\sqcup$H$_A$)) =(F$_A\sqcup$H$_A$) - (ϕ_G(F$_A$)$\sqcup\phi_G$(H$_A$))$\sqsubseteq$ (F$_A$ - ϕ_G(F$_A$))$\sqcup$(H$_A$ - ϕ_G(H$_A$))$\notin$G. Hence, F$_A\sqcup$H$_A$ is GL-perfect soft set.

(ii) Let F_A, H_A be GR-perfect soft sets. Then, $(\phi_G(F_A)$ - $F_A)\notin$G and $(\phi_G(H_A)$ - $H_A)\notin$G. By the definition of soft grill, $(\phi_G(F_A)$ - $F_A)\sqcup(\phi_G(H_A)$ - $H_A)\notin$G. Since, $(\phi_G(F_A)\sqcup\phi_G(H_A))$ - $(F_A\sqcup H_A)\sqsubseteq(\phi_G(F_A)$ - $F_A)\sqcup(\phi_G(H_A)$ - $H_A)$, then $(\phi_G(F_A\sqcup H_A)$ - $(F_A\sqcup H_A))=(\phi_G(F_A)\sqcup\phi_G(H_A))$ - $(F_A\sqcup H_A)\notin$G. Hence, $F_A\sqcup H_A$ is GR-soft perfect set.

Corollary 4.1 The soft union of two Gϱ-perfect soft sets F_A, H_A is Gϱ-perfect soft set. **Proof**: Straightforward.

Theorem 4.5 The soft intersection of two GR-perfect soft sets F_A, H_A is GR-perfect soft set. Let G be a soft grill on a soft topological space (X, τ, A). If F_A, H_A are GR-perfect soft sets, then $F_A\sqcap H_A$ is GR-perfect soft set.

Proof: Let F_A, H_A be GR-perfect soft sets. Then, $(\phi_G(F_A)$ - $F_A)\notin$G and $(\phi_G(H_A)$ - $H_A)\notin$G. So $(\phi_G(F_A)\sqcap\phi_G(H_A)$ - $F_A)\notin$G and $(\phi_G(F_A)\sqcap\phi_G(H_A)$ - $H_A)\notin$G. By the definition of soft grill, $(\phi_G(F_A)\sqcap\phi_G(H_A)$ - $F_A)\sqcup(\phi_G(F_A)\sqcap\phi_G(H_A)$ - $H_A)\notin$G. Thus $(\phi_G(F_A\sqcap H_A)$ - $(F_A\sqcap H_A))\sqsubseteq(\phi_G(F_A)\sqcap\phi_G(H_A))$ - $(F_A\sqcap H_A)\notin$G. Hence, $(\phi_G(F_A\sqcap H_A)$ - $(F_A\sqcap H_A))\notin$G and so $F_A\sqcap H_A$ is GR-perfect soft set.

Corollary 4.2 Let G be a soft grill on a soft topological space (X, τ, A) and F_A, $H_A\in\mathcal{SS}$(X, A). Then,

(i) Finite union of GL-perfect soft sets is an GL-perfect soft set.

(ii) Finite union of GR-perfect soft sets is an GR-perfect soft set.

(iii) Finite union of Gϱ-perfect soft sets is an Gϱ-perfect soft set.

(iv) Finite intersection of GR-perfect soft sets is an GR-perfect soft set.

Theorem 4.5 Let G be a soft grill on a soft topological space (X, τ, A) with X is finite, the collection of soft sets GR-$\mathcal{PSS}$(X, A)$\sqsubseteq\mathcal{SS}$(X, A) is finite soft topology, which is finer than soft topology τ_G.

Proof: It is follows from Lemma 4.2, Lemma 4.5 and Corollary 4.2.

The collection of soft sets GR-$\mathcal{PSS}$(X, A) is a soft topology on a finite initial universe set X follows from (i), (ii) of Lemma 3.7 and (ii), (iv) of Corollary 3.15. In view of (ii) of Lemma 3.3, the collection of soft sets GR-$\mathcal{PSS}$(X, A) is finer than soft topology τ_G.

Definition 4.3 Let $\wp$ be the collection of soft sets over X with the fixed set of parameters A. Then, $\wp$ is called a generalized soft topology on X, if it satisfies the following axioms:

(i) $\tilde{\emptyset}_A$ belongs to $\wp$.

(ii) the union of any number of soft sets in $\wp$ belongs to $\wp$.

The triplet (X, A, $\wp$) is called a generalized soft topological space over X. Every member of $\wp$ is called an $\wp$-open soft set. The complement of an $\wp$-open soft set is called the $\wp$-closed soft set.

Theorem 4.6 Let X be finite set, then for any grill G the collection GL-$\mathcal{PSS}$(X, A) is a finite generalized soft topology.

Proof: It is follows directly from Lemma 4.5 and Corollary 4.2.

Follows directly according to (i) of Lemma 3.7 and (i) of Corollary 3.15.

Theorem 4.7 Let G be a soft grill on a soft topological space (X, τ, A). If the soft topology τ is suitable for a soft G on X, then GL-$\mathcal{PSS}$(X, A)=$\mathcal{SS}$(X, A).

Proof: Obvious by Theorem 3.9.

Theorem 4.8 Let G be a soft grill on a soft topological space (X, τ, A) such that the soft topology τ is suitable for a soft G and $F_A \in \mathcal{SS}$(X, A). Then, (i) If $F_A \sqcap \phi_G(F_A) = \widetilde{\emptyset}_A$, then F_A is Gϱ-perfect soft set.
(ii) (F_A - $\phi_G(F_A)$) is Gϱ-perfect soft set.
(iii) $x_e \in F_A$ there exists an open soft nbd. U_A of x_e with $U_A \sqcap F_A \notin$G, it follows that F_A is Gϱ-perfect soft set.

Proof: Follows directly from Lemma 4.2 and Theorem 4.7.

Theorem 4.9 Let G be a soft grill on a soft topological space (X, τ, A) and $F_A \in \mathcal{SS}$(X, A), then the following statements are equivalent
(i) The soft topology τ is suitable for a soft grill G.
(ii) If F_A is GR-perfect soft set, then $F_A \Delta \phi_G(F_A) \notin$G.

Proof: (i)$\Longrightarrow$(ii) Let F_A be GR-perfect soft set, then ($\phi_G(F_A)$ - F_A)$\notin$G. In view of Theorem 3.19 and (i) of this Theorem (F_A - $\phi_G(F_A)$)$\notin$G and ($\phi_G(F_A)$ - F_A)$\notin$G. Hence, $F_A \Delta \phi_G(F_A) \notin$G.
(ii)$\Longrightarrow$(i) Since (F_A - $\phi_G(F_A)$)$\sqsubseteq F_A \Delta \phi_G(F_A)$, then by (ii) ($F_A$ - $\phi_G(F_A)$)$\notin$G. Hence, The soft topology τ is suitable for a soft grill G.

References

1 Ali, M. I.; Feng, F.; Liu, X.; Min, W. K. and Shabir, M. *On some new operations in soft set theory*, Comput. Math. Appl., 57, 1547–1553, (2009).
2 Aygunoglu, A.; and Aygun, H.; *Some notes on soft topological spaces*, Neural. Comput. Appl., 1–7, (2011).
3 Cagman, N.; Karatas, S.; and Enginoglu, S.; *Soft topology*, Comput. Math. Appl., 62, 351–358, (2011).
4 Choquet, G.; *Sur les notions de filtre et grille*, Comptes Rendus Acad. Sci. Paris, 224, (1947), 171–173.
5 Csaszar, A.; *Generalized topology, generalized continuity*, Acta Math. Hungar.,96, 351–357, (2002).
6 Hosny, Rodyna A.; Remarks on soft topological spaces with soft grill far East J. Math. Sci., 86(1), 111–128, (2014).
7 Hosny, Rodyna A.; El-Kadi, Deena; Soft semi open sets with respect to soft ideals, Applied Math. Sci., 8 (150),

7487–7501, (2014).

8 Hussain, S.; and Ahmad, B.; *Some properties of soft topological spaces*, Comput. Math. Appl., 62, 4058–4067, (2011).

9 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; $\gamma-$operation and decompositions of some forms of soft continuity of soft topological spaces via soft ideal, Ann. Fuzzy Math. Inform., 9 (3) (2015) 385-402.

10 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft connectedness via soft ideals, Journal of New Results in Science, 4 (2014) 90–108.

11 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft ideal theory, Soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014) 1595–1603.

12 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft regularity and normality based on semi open soft sets and soft ideals, Appl. Math. Inf. Sci. Lett., 3 (2) (2015) 47–55.

13 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft semi compactness via soft ideals, Appl. Math. Inf. Sci., 8 (5) (2014) 2297–2306.

14 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Soft semi (quasi) Hausdorff spaces via soft ideals, South Asian J. Math., 4 (6) (2014) 265–284.

15 Kandil, A.; Tantawy, O. A. E.; El-Sheikh, S. A.; and Abd El-latif, A. M.; Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces, Appl. Math. Inf. Sci., 8 (4) (2014) 1731-1740.

16 Maji, P. K.; Biswas, R. and Roy, A. R.; *Soft set theory*, Compu. and Math. Appl., 5, 555–562, (2003).

17 Min, W. K.; *A note on soft topological spaces*, Comput. Math. Appl., 62, 3524–3528, (2011).

18 Molodsov, D. *Soft set theory-first results*, Compu. and Math. Appl., 37, 19–31, (1999).

19 Nazmul, Sk.; Samanta, S. K.; *Neighbourhood properties of soft topological spaces*, Annals of Fuzzy Mathematics and Informatics spaces, 6(1), 1–15, (2013).

20 Shabir, M. and Naz, M.; *On soft topological spaces*, Comput. Math. Appl., 61, 1786–1799, (2011).

21 Zorlutuna, I.; Akdag, M.; Min, W. K.; and Atmaca, S.; *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 3, 171–185, (2012).