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Fuzzy soft topological spaces: regularity and separation axioms

A. Kandil[①*], O.A.E. Tantawy[②], S.A. El-Sheikh[③], Sawsan S. S. El-Sayed[③④]

① Mathematics Department, Faculty of Science, Helwan University, Helwan, Egypt
② Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt
③ Mathematics Department, Faculty of Education, Ain Shams University, Cairo, Egypt
④ Mathematics Department, Faculty of Education, Majmaah University, Majmaah, Kingdom of Saudi Arabia
E-mail: s.elsayed@mu.edu.sa

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Abstract In this paper, regularity and separation axioms in fuzzy soft topological spaces are defined and studied by using quasi-coincident relation and fuzzy soft neighborhood system. We discuss its charaterizations and relationship among them. In addition, goodness and hereditary properties are discussed.

Key Words Fuzzy soft set, fuzzy soft topological space, fuzzy soft R_i-spaces, and fuzzy soft T_i-spaces
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1 Introduction

We are not able to use classical methods to solve some kinds of problems given in sociology, economics, environment, engineering etc., since, these kinds of problems have their own uncertainties. Fuzzy set theory, which was firstly proposed by Zadeh [29] in 1965, has become a very important tool to solve these kinds of problems and provides an appropriate framework for representing vague concepts by allowing partial membership. Fuzzy set theory has been studied by both mathematicians and computer scientists and many applications of fuzzy set theory have arisen over the years, such as fuzzy control systems, fuzzy automata, fuzzy logic, fuzzy topology etc. Beside this theory, there are also theory of probability, rough set theory which deal with to solve these problems. Each of these theories has its inherent difficulties as pointed out in 1999 by Molodtsov [23] who introduced the concept of soft set theory which is a completely new approach for modeling uncertainty. In this paper, Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Maji et al. [21] defined and studied several basic notions of soft set theory in 2003. Shabir and Naz [26]

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introduced the concept of soft topological space and studied neighborhoods and separation axioms. Maji et al. [22] initiated the study involving both fuzzy sets and soft sets. In this paper, the notion of fuzzy soft sets was introduced as a fuzzy generalizations of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Maji et al. combined fuzzy sets and soft sets and introduced the concept of fuzzy soft sets. To continue the investigation on fuzzy soft sets, Ahmad and Kharal [1] presented some more properties of fuzzy soft sets and introduced the notion of a mapping on fuzzy soft sets. In 2011, Tanay et al. [27] gave the topological structure of fuzzy soft sets. Kandil et al. introduced the concept of fuzzy soft connected sets [16, 17, 18], fuzzy soft hyperconnected spaces [19] and fuzzy soft ideal theory [14, 15].

The concept of separation axioms is one of the most important concepts in topological spaces. In fuzzy setting, it had been studied by many authors such as: Das [5], Saha [6], Hutton [8, 9], and Kandil [10, 11, 12]. In soft setting, it has been studied by Shabir [26] and Göçür [7] et. al.. In fuzzy soft setting, it had been studied by Mahanta [20]. The object of the present paper is to inrroduce a set of new regularity and separation axioms which are called (FSR_i; $i = 0, 1, 2, 3$) and (FST_i; $i = 0, 1, 2, 3, 4$) by using fuzzy soft quasi-coincident and neighborhood system.

In the present study we consider the topological structure of fuzzy soft set theory. Firstly, as a preliminaries, we give some basic definitions and results in fuzzy soft set theory. After giving these preliminaries, we define the notion of fuzzy soft regularity axioms (FSR_i; $i = 0, 1, 2, 3$) and separation axioms (FST_i; $i = 0, 1, 2, 3, 4$). Finally, the notion of fuzzy soft hereditary property is examined.

2 Preliminaries

Throughout this paper X denotes initial universe, E denotes the set of all possible parameters which are attributes,characteristic or properties of the objects in X, and the set of all subsets of X will be denoted by $P(X)$. In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1. [4] A fuzzy set A of a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0, 1] = I$ whose value $\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$. Let I^X denotes the family of all fuzzy sets on X.

Definition 2.2. [23] Let A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e -approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e $F = \{F(e) : e \in A \subseteq E, F : A \to P(X)\}$.

Proposition 2.1. [2] Every fuzzy set may be considered a soft set.

Definition 2.3. [22] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called fuzzy soft set over X, where f is a mapping given by $f : A \longrightarrow I^X$ defined by $f_A(e) = \mu^e_{f_A}$; where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$, and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$, where $\overline{0}(x) = 0 \ \forall \ x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_E$. Note that, a fuzzy soft set is a hybridizition of fuzzy sets and soft sets, in which soft set is defined over fuzzy set.

Definition 2.4. [27] The complement of a fuzzy soft set (f,A), denoted by $(f,A)^c$, is defined by $(f,A)^c$ $=(f^c,A)$, $f_A^c : E \longrightarrow I^X$ is a mapping given by $\mu^e_{f_A^c} = \overline{1} - \mu^e_{f_A}$ $\forall e \in A$, where $\overline{1}(x) = 1 \forall x \in X$. Clearly, $(f_A^c)^c = f_A$.

Definition 2.5. [27] A fuzzy soft set f_E over X is said to be a null- fuzzy soft set, denoted by $\widetilde{0}_E$, if for all $e \in E$, $f_E(e) = \overline{0}$.

Definition 2.6. [27] A fuzzy soft set f_E over X is said to be an absolute fuzzy soft set, denoted by $\widetilde{1}_E$, if $f_E(e) = \overline{1}$ $\forall e \in E$. Clearly we have $(\widetilde{0}_E)^c = \widetilde{1}_E$ and $(\widetilde{1}_E)^c = \widetilde{0}_E$.

Definition 2.7. [27] Let $f_A \in FSS(X)_E$. The fuzzy soft set f_A is called the A-universal fuzzy soft set, denoted by $\widetilde{1}_A$, if for all $e \in A$, $f_A(e) = \overline{1}$ and $f_A(e) = \overline{0}, \forall e \in E \backslash A$.

Definition 2.8. [22, 24, 25, 27, 28] Let f_A and $g_B \in FSS(X)_E$. Then f_A is fuzzy soft subset of g_B, denoted by $f_A \subseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A}(x) \leqslant \mu^e_{g_B}(x)$ $\forall x \in X, \forall e \in E$. Also, g_B is called fuzzy soft superset of f_A denoted by $g_B \supseteq f_A$. If f_A is not fuzzy soft subset of g_B, we write $f_A \nsubseteq g_B$

Definition.2.9. [22, 24, 25, 27, 28] Two fuzzy soft sets f_A and g_B on X are called equal if $f_A \subseteq g_B$ and $g_B \subseteq f_A$.

Definition 2.10. [22, 24, 25, 27, 28] The union of two fuzzy soft sets f_A and g_B over the common universe X, denoted by $f_A \sqcup g_B$, is also a fuzzy soft set h_C, where $C = A \cup B$ and for all, $e \in C$, $h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \vee \mu^e_{g_B}$ $\forall e \in E$.

Definition 2.11. [22, 24, 25, 27, 28] The intersection of two fuzzy soft sets f_A and g_B over the common universe X, denoted by $f_A \sqcap g_B$, is also a fuzzy soft set h_C, where $C = A \cap B$ and for all, $e \in C, h_C(e) = \mu^e_{h_C} = \mu^e_{f_A} \wedge \mu^e_{g_B}$ $\forall e \in E$.

Definition 2.12. [27] Let τ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E , then τ is called fuzzy soft topology on X if

(1) $\widetilde{0}_E$, $\widetilde{1}_E \in \tau$, where $\widetilde{0}_E(e) = \overline{0}$ and $\widetilde{1}_E(e) = \overline{1}$ $\forall e \in E$,

(2) the union of any members of τ belongs to τ.

(3) the intersection of any two members of τ belongs to τ.

The triplet (X, τ, E) is called fuzzy soft topological space over X. Also, each member of τ is called fuzzy soft open in (X, τ, E).

Examples 2.1. The following are fuzzy soft topology on X :

(1) $\tau_0 = \{\widetilde{0}_E, \widetilde{1}_E\}$ is called fuzzy soft indiscrete topology on X.

(2) $\tau_D = FSS(X)_E$ is called fuzzy soft discrete topology on X.

Note that, the intersection of any family of fuzzy soft topologies on X is also a fuzzy soft topology on X.

Definition 2.13. [27] Let (X, τ, E) be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, denoted by $f_A \in \tau^c$, if its relative complement f_A^c is fuzzy open soft set. Clearly, $\widetilde{0}_E$ and $\widetilde{1}_E$ are fuzzy soft closed sets, arbitrary intersection of fuzzy soft closed sets is a fuzzy soft closed, and the finite union of fuzzy soft closed sets is a fuzzy soft closed.

Definition 2.14. [24, 25] Let (X, τ, E) be a fuzzy soft topological space and $f_A \in FSS(X)_E$ .The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A, i.e. $Fcl(f_A) = \sqcap\{h_C; h_C$ is fuzzy closed soft set and $f_A \subseteq h_C\}$. Clearly, $Fcl(f_A)$ is the smallest fuzzy soft closed set over X which contains f_A, and $Fcl(f_A)$ is fuzzy closed soft set.

Definition 2.15. [25, 28] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$; $(0 \leqslant \alpha \leqslant 1)$ and $\mu^e_{f_A}(y) = 0\ \forall y \in X - \{x\}$ and this fuzzy soft point is denoted by x^e_α or f_e . The class of all fuzzy soft points of X, denoted by $FSP(X)_E$.

Definition 2.16. [20] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set f_A, denoted by $x^e_\alpha \widetilde{\in} f_A$, if for the element $e \in A$, $\alpha \leqslant \mu^e_{f_A}(x)$.

Definition 2.17. [25, 28] A fuzzy soft point x^e_α is said to be a quasi-coincident with a fuzzy soft set f_A, denoted by $x^e_\alpha\ q\ f_A$, if $\alpha + \mu^e_{f_A}(x) > 1$. The negation of this statment is written as $x^e_\alpha\ \overline{q}\ f_A$.

Definition 2.18. [25, 28] A fuzzy soft set f_A is said to be quasi-coincident with g_B, denoted by $f_A\ q\ g_B$, if there exists $x \in X$ such that $\mu^e_{f_A}(x) + \mu^e_{g_B}(x) > 1$, for some $e \in A \cap B$. If this is true we can say that f_A and g_B are quasi-coincident at x.

Definition 2.19. [25, 28] Let (X, τ, E) be a fuzzy soft topological space and x^e_α be a fuzzy soft point in X. A fuzzy soft set f_A is called fuzzy soft q-neighborhood of x^e_α (fuzzy soft q-nbd, for short), if ther exists $g_B \in \tau$ such that $x^e_\alpha\ q\ g_B$ and $g_B \subseteq f_A$.

Proposition 2.2. [25] Let $N(x^e_\alpha)$ be the family of all fuzzy soft q-nbd of x^e_α in a fuzzy soft topological space (X, τ, E). The following holds:

(i) If $f_A \in N(x^e_\alpha)$, then $x^e_\alpha\ q\ f_A$,

(ii) If $f_A \in N(x^e_\alpha)$ and $f_A \subseteq g_B$, then $g_B \in N(x^e_\alpha)$,

(iii) If f_A, $g_B \in N(x^e_\alpha)$, then $f_A \sqcap g_B \in N(x^e_\alpha)$,

(iv) If $f_A \in N(x^e_\alpha)$, then there exists $g_B \in N(x^e_\alpha)$ such that $g_B \subseteq f_A$ and $g_B \in N(y^e_\beta)$ for every fuzzy soft point y^e_β which is quasi-coincident with g_B.

Proposition 2.3. Let f_A, g_B, $h_C \in FSS(X)_E$ and x^e_α, $y^t_\beta \widetilde{\in} FSP(X)_E$; $0 \leqslant \alpha, \beta \leqslant 1$, $e, t \in E$. Then:

(1) $f_A\ \overline{q}\ g_B \Longleftrightarrow f_A \subseteq g^c_B$,

(2) $f_A\ \overline{q}\ g_B \Longleftrightarrow g_B\ \overline{q}\ f_A$,

(3) $f_A \sqcap g_B = \widetilde{0}_E \Longrightarrow f_A\ \overline{q}\ g_B$,

(4) $f_A\ q\ g_B \Longrightarrow f_A \sqcap g_B \neq \widetilde{0}_E$,

(5) $f_A\ \overline{q}\ f^c_A$,

(6) $f_A\ \overline{q}\ g_B$, $h_C \subseteq g_B \Longrightarrow f_A\ \overline{q}\ h_C$,

(7) $f_A \subseteq g_B \Longleftrightarrow (x^e_\alpha\ q\ f_A \Longrightarrow x^e_\alpha\ q\ g_B)$; $x^e_\alpha \widetilde{\in} FSP(X)_E$ or $(x^e_\alpha\ \overline{q}\ g_B \Longrightarrow x^e_\alpha\ q\ f_A)$,

(8) $x^e_\alpha\ q\ (\bigsqcup\limits_{i \in J} (g_B)_i) \Longleftrightarrow x^e_\alpha\ q\ (g_B)_{i_0}$ for some $i_0 \in J$,

(9) $x^e_\alpha\ q\ (f_A \sqcap g_B) \Longleftrightarrow (x^e_\alpha\ q\ f_A$ and $x^e_\alpha\ q\ g_B)$,

(10) $x \neq y \Longrightarrow x^e_\alpha\ \overline{q}\ y^t_\beta\ \forall 0 < \alpha,\ \beta < 1$ and $\forall e,\ t \in E$,

(11) $x^e_\alpha\ \overline{q}\ y^t_\beta \Longleftrightarrow x \neq y$ or $(x = y,\ e = t$, but $\alpha + \beta \leqslant 1)$, or $(x = y,\ \alpha + \beta > 1$, but $e \neq t)$.

Proposition 2.4. Let (X, τ, E) be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $x^e_\alpha \widetilde{\in} FSP(X)_E$. Then we have:

(1) $[Fint(f_A)]^c = Fcl(f^c_A)$,

(2) $x^e_\alpha \widetilde{\in} Fint(f_A) \Longleftrightarrow \exists O_{x^e_\alpha} \in N(x^e_\alpha)$ such that $O_{x^e_\alpha} \subseteq f_A$,

(3) $x^e_\alpha\ q\ Fcl(f_A) \Longleftrightarrow O_{x^e_\alpha}\ q\ f_A\ \forall O_{x^e_\alpha} \in N(x^e_\alpha)$,

(4) $g_B\ q\ f_A \Longleftrightarrow g_B\ q\ Fcl(f_A)\ \forall g_B \in \tau$.

Definition 2.20. [13] Let $(f_A,\ \tau_{f_A},\ A)$ be a fuzzy soft topological space and g_B be a fuzzy soft subset of f_A. Then $\tau_{g_B} = \{h_C \sqcap g_B; h_C \in \tau_{f_A}\}$ is called fuzzy soft relative topology and $(g_B,\ \tau_{g_B},\ B)$ is called fuzzy soft subspace. If $g_B \in \tau_{f_A}$, then $(g_B,\ \tau_{g_B},\ B)$ is called fuzzy soft open subspace. If $g_B \in \tau^c_{f_A}$, then (g_B, τ_{g_B}, B) is called fuzzy soft closed subspace.

3 Fuzzy soft regularity axioms

Definition 3.1. A fuzzy soft topological spaces (X, τ, E) is said to be:

(1) fuzzy soft R_0-space (FSR_0-space for short) if $\forall x^e_\alpha, y^t_\beta \widetilde{\in} FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta) \Longrightarrow Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$.

(2) fuzzy soft R_1-space (FSR_1-space for short) if $\forall x^e_\alpha, y^t_\beta \widetilde{\in} FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ implies $\exists O_{x^e_\alpha} \in N(x^e_\alpha)$ and $O_{y^t_\beta} \in N(y^t_\beta)$ such that $O_{x^e_\alpha}\ \overline{q}\ O_{y^t_\beta}$.

(3) fuzzy soft R_2-space (FSR_2-space for short) if $\forall x^e_\alpha \widetilde{\in} FSS(X)_E$ and $\forall g_B \in \tau^c$ with $x^e_\alpha\ \overline{q}\ g_B$ implies $\exists O_{x^e_\alpha} \in N(x^e_\alpha)$ and $O_{g_B} \in N(g_B)$ such that $O_{x^e_\alpha}\ \overline{q}\ O_{g_B}$.

(4) fuzzy soft R_3-space (FSR_3-space for short) if $\forall f_A,\ g_B \in \tau^c$ with $f_A\ \overline{q}\ g_B$ implies $\exists O_{f_A} \in N(f_A)$ and $O_{g_B} \in N(g_B)$ such that $O_{f_A}\ \overline{q}\ O_{g_B}$.

Theorem 3.1. Let (X, τ, E) be a fuzzy soft topological space, $x^e_\alpha, y^t_\beta \widetilde{\in} FSP(X)_E$ and $f_A \in \tau^c$. The following statements are equivalent:

(1) (X, τ, E) is a FSR_0-space,

(2) $x^e_\alpha\ q\ Fcl(y^t_\beta) \Longrightarrow Fcl(x^e_\alpha)\ q\ y^t_\beta$,

(3) $Fcl(x^e_\alpha) \subseteq O_{x^e_\alpha}\ \forall O_{x^e_\alpha} \in N(x^e_\alpha)$,

(4) $Fcl(x^e_\alpha) \subseteq \sqcap\{O_{x^e_\alpha}\ ; O_{x^e_\alpha} \in N(x^e_\alpha)\}$,

(5) $x^e_\alpha\ \overline{q}\ f_A$ implies $\exists O_{f_A} \in N(f_A)$ such that $x^e_\alpha\ \overline{q}\ O_{f_A}$,

(6) $x^e_\alpha\ \overline{q}\ f_A$ implies $Fcl(x^e_\alpha)\ \overline{q}\ f_A$,

(7) $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies $Fcl(x^e_\alpha)\ \overline{q}\ Fcl(y^t_\beta)$.

Proof. (1)$\Longrightarrow$ (2): Let $x^e_\alpha\ q\ Fcl(y^t_\beta)$. Suppose $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$. Since (X, τ, E) is a FSR_0-space, then $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ which is a contradicton. Therefore, $Fcl(x^e_\alpha)\ q\ y^t_\beta$.

(2) $\Longrightarrow$ (3): Let $y^t_\beta\ q\ Fcl(x^e_\alpha)$.By (2), then $x^e_\alpha\ q\ Fcl(y^t_\beta)$. By (3) of Proposition 2.4, we have $y^t_\beta\ q\ O_{x^e_\alpha}\ \forall O_{x^e_\alpha} \in N(x^e_\alpha)$. Therefore, $Fcl(x^e_\alpha) \subseteq O_{x^e_\alpha} \forall O_{x^e_\alpha} \in N(x^e_\alpha)$ (by (6) of Proposition 2.3).

(3) $\Longrightarrow$ (4): Obvious.

(4) $\Longrightarrow$ (5): Let $x^e_\alpha\ \overline{q}\ f_A$. Then $x^e_\alpha \widetilde{\in} f^c_A$. By (3), $Fcl(x^e_\alpha) \subseteq f^c_A$ and so $f_A \subseteq [Fcl(x^e_\alpha)]^c = O_{f_A}$. Thus $x^e_\alpha\ \overline{q}\ [Fcl(x^e_\alpha)]^c = O_{f_A}$.

(5) $\Longrightarrow$ (6): Let $x^e_\alpha \ \overline{q}\ f_A$. By (5), there exists O_{f_A} such that $x^e_\alpha \ \overline{q}\ O_{f_A}$. Then $x^e_\alpha \ \widetilde{\in}\ O^c_{f_A}$ and so $Fcl(x^e_\alpha) \subseteq O^c_{f_A}$. Therefore, $Fcl(x^e_\alpha) \ \overline{q}\ O_{f_A}$. By (5) of Proposition 2.3, $Fcl(x^e_\alpha) \ \overline{q}\ f_A$.

(6) $\Longrightarrow$ (7) and (7) $\Longrightarrow$ (1) are obvious.

Theorem 3.2. The following implications hold:

$$\mathrm{FSR}_3 \wedge \mathrm{FSR}_0 \Longrightarrow \mathrm{FSR}_2 \Longrightarrow \mathrm{FSR}_1 \Longrightarrow \mathrm{FSR}_0$$

Proof. (i) $\mathrm{FSR}_3 \wedge \mathrm{FSR}_0 \Longrightarrow \mathrm{FSR}_2$: Let (X,τ,E) be $\mathrm{FSR}_3 \wedge \mathrm{FSR}_0$-space and $x^e_\alpha \ \overline{q}\ g_B$ such that $g_B \in \tau^c$. By (6) of Theorem 3.1, we have $Fcl(x^e_\alpha) \ \overline{q}\ g_B$. Since (X,τ,E) is FSR_3-space, then there exist $O_{Fcl(x^e_\alpha)}$ and O_{g_B} such that $O_{Fcl(x^e_\alpha)} \ \overline{q}\ O_{g_B}$. Take $O_{x^e_\alpha} = O_{Fcl(x^e_\alpha)}$, then $O_{x^e_\alpha} \overline{q}\ O_{g_B}$ and hence (X,τ,E) is a FSR_2-space.

(ii) $\mathrm{FSR}_2 \Longrightarrow \mathrm{FSR}_1$: Let (X,τ,E) be FSR_2-space and $x^e_\alpha \ \overline{q}\ Fcl(y^t_\beta)$. Then there exist $O_{x^\alpha_e}$ and $O_{Fcl(y^t_\beta)} \in \tau$ such that $O_{x^\alpha_e} \ \overline{q}\ O_{Fcl(y^t_\beta)}$. Take $O_{y^t_\beta} = O_{Fcl(y^t_\beta)}$. Then $O_{x^\alpha_e} \ \overline{q}\ O_{y^t_\beta}$ and hence (X,τ,E) is a FSR_1-space.

(iii) $\mathrm{FSR}_1 \Longrightarrow \mathrm{FSR}_0$: Let (X,τ,E) be FSR_1-space and $x^e_\alpha \ \overline{q}\ Fcl(y^t_\beta)$. Then there exist $O_{x^e_\alpha}$ and $O_{y^t_\beta} \in \tau$ such that $O_{x^e_\alpha} \ \overline{q}\ O_{y^t_\beta}$. Thus $x^e_\alpha \ \overline{q}\ O_{y^t_\beta}$. By (3) of Proposition 2.4, we have $y^t_\beta \ \overline{q}\ Fcl(x^e_\alpha)$. Hence (X,τ,E) is a FSR_0-space.

Corollary 3.1. Let (X,τ,E) be a fuzzy soft topological space. Then (X,τ,E) is a FSR_1-space if and only if $\forall x^e_\alpha,\ y^t_\beta \in FSP(X)_E$ with $x^e_\alpha \ \overline{q}\ Fcl(y^t_\beta)$ implies there exist $O_{Fcl(x^e_\alpha)}$ and $O_{Fcl(y^t_\beta)} \in \tau$ such that $O_{Fcl(x^e_\alpha)} \ \overline{q}\ O_{Fcl(y^t_\beta)}$.

Proof. Follows directly from Theorem 3.2 and Therorem 3.1 (3).

Theorem 3.3. Let (X,τ,E) be a fuzzy soft topological space. Then (X,τ,E) is a FSR_2-space if and only if for all $x^e_\alpha \in FSP(X)_E$ and for all $O_{x^e_\alpha} \in N(x^e_\alpha)$, there exists $O^*_{x^e_\alpha}$ such that $Fcl(O^*_{x^e_\alpha}) \subseteq O_{x^e_\alpha}$.

Proof. Let (X,τ,E) be a FSR_2-space, $x^e_\alpha \in FSP(X)_E$ and $O_{x^e_\alpha} \in N(x^e_\alpha)$. Then $x^e_\alpha \ \overline{q}\ O^c_{x^e_\alpha}$. Therefore, there exist $O^*_{x^e_\alpha} \in N(x^e_\alpha)$ and $g_B \in N(O^c_{x^e_\alpha})$ such that $O^*_{x^e_\alpha} \ \overline{q}\ g_B$. This implies that $O^*_{x^e_\alpha} \subseteq g^c_B \in \tau^c$. Thus $Fcl(O^*_{x^e_\alpha}) \subseteq g^c_B \subseteq O_{x^e_\alpha}$.

Conversely, let $x^e_\alpha \in FSP(X)_E$ and $g_B \in \tau^c$ be such that $x^e_\alpha \ \overline{q}\ g_B$. Then $x^e_\alpha \ \widetilde{\in}\ g^c_B$ i.e. $g^c_B \in N(x^e_\alpha)$, so there exists $O^*_{x^e_\alpha}$ such that $Fcl(O^*_{x^e_\alpha}) \subseteq g^c_B$. Thus $g_B \subseteq [Fcl(O^*_{x^e_\alpha})]^c$. Take $O_{g_B} = [Fcl(O^*_{x^e_\alpha})]^c$. Therefore, $O_{g_B} \ \overline{q}\ O^*_{x^e_\alpha}$. Hence, (X,τ,E) is a FSR_2-space.

Theorem 3.4. Let (X,τ,E) be a fuzzy soft topological space. Then (X,τ,E) is a FSR_3-space if and only if for all $f_A \in \tau^c$ and for all O_{f_A} there exists $O^*_{f_A}$ such that $Fcl(O^*_{f_A}) \subseteq O_{f_A}$.

Proof. Proof manner similar to the proof of the previous theorem.

4 Fuzzy soft separation axioms

Definition 4.1. A fuzzy soft topological spaces (X,τ,E) is said to be:

(1) fuzzy soft T_0-space (FST_0-space for short) if $\forall x^e_\alpha,\ y^t_\beta \in FSP(X)_E$ with $x^e_\alpha \ \overline{q}\ y^t_\beta$ implies there exists $O_{x^e_\alpha} \in N(x^\alpha_e)$ such that $O_{x^e_\alpha} \ \overline{q}\ y^t_\beta$ or there exists $O_{y^t_\beta} \in N(y^t_\beta)$ such that $O_{y^t_\beta} \ \overline{q}\ x^e_\alpha$.

(2) fuzzy soft T_1-space (FST_1-space for short) if if $\forall x^e_\alpha,\ y^t_\beta \in FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies there exists $O_{x^e_\alpha} \in N(x^\alpha_e)$ such that $O_{x^e_\alpha}\ \overline{q}\ y^t_\beta$ and there exists $O_{y^t_\beta} \in N(y^t_\beta)$ such that $O_{y^t_\beta}\ \overline{q}\ x^e_\alpha$.

(3) fuzzy soft T_2-space (FST_2-space for short) if if $\forall x^e_\alpha,\ y^t_\beta \in FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies there exist $O_{x^e_\alpha} \in N(x^\alpha_e)$ and $O_{y^t_\beta} \in N(y^t_\beta)$ such that $O_{x^e_\alpha}\ \overline{q}\ O_{y^t_\beta}$.

(4) fuzzy soft T_3-space (FST_3-space for short) if it is FSR_2 and FST_1-space.

(5) fuzzy soft T_4-space (FST_4-space for short) if it is FSR_3 and FST_1-space.

Theorem 4.1. Let (X, τ, E) be a fuzzy soft topological space. Then (X, τ, E) is a FST_0-space if and only if $\forall x^e_\alpha,\ y^t_\beta \in FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ or $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$.

Proof. Let (X, τ, E) be a FST_0-space and $x^e_\alpha\ \widetilde{q}\ y^t_\beta$. Then there exist $O_{x^e_\alpha}$ such that $O_{x^e_\alpha}\ \overline{q}\ y^t_\beta$ or there exist $O_{y^t_\beta}$ such that $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$. By (3) of Proposition 2.4, we have $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ or $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$. Conversely, let $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ or $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$. Then $x^e_\alpha\ \widetilde{\in}\ [Fcl(y^t_\beta)]^c$ or $y^t_\beta\ \widetilde{\in}\ [Fcl(x^e_\alpha)]^c$. Take $O_{x^e_\alpha} = [Fcl(y^t_\beta)]^c$ and $O_{y^t_\beta} = [Fcl(x^e_\alpha)]^c$. Therefore, $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$ or $y^t_\beta\ \overline{q}\ O_{x^e_\alpha}$. Hence, (X, τ, E) is a FST_0-space.

Theorem 4.2. Let (X, τ, E) be a fuzzy soft topological space. The following are equivalent:

(1) (X, τ, E) is a FST_1-space,

(2) $\forall x^e_\alpha, y^t_\beta \in FSP(X)_E$ with $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ and $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$,

(3) $Fcl(x^e_\alpha) = x^e_\alpha\ \forall x^e_\alpha \in FSP(X)_E$.

Proof. (1) $\Leftrightarrow$ (2): Let (X, τ, E) be a FST_1-space and $x^e_\alpha\ \overline{q}\ y^t_\beta$. Then there exist $O_{x^e_\alpha}$ and $O_{y^t_\beta}$ such that $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$ and $y^t_\beta\ \overline{q}\ O_{x^e_\alpha}$. By (3) of Proposition 2.4, we have $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ and $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$.

conversely, let $x^e_\alpha\ \overline{q}\ y^t_\beta$ implies $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$ and $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$. Then $x^e_\alpha\ \widetilde{\in}\ [Fcl(y^t_\beta)]^c$ and $y^t_\beta\ \widetilde{\in}\ [Fcl(x^e_\alpha)]^c$. Take $O_{x^e_\alpha} = [Fcl(y^t_\beta)]^c$ and $O_{y^t_\beta} = [Fcl(x^e_\alpha)]^c$. Therefore, $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$ and $y^t_\beta\ \overline{q}\ O_{x^e_\alpha}$. Hence, (X, τ, E) is a FST_1-space.

(1) $\Longrightarrow$ (3): Let $x^e_\alpha\ \overline{q}\ y^t_\beta$. Then there exist $O_{y^t_\beta}$ such that $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$. This implies $O_{y^t_\beta} \subseteq (x^e_\alpha)^c$. Thus $(x^\alpha_e)^c$ is a fuzzy soft open i.e. x^e_α is a fuzzy soft closed. Hence $Fcl(x^e_\alpha) = x^e_\alpha$ and this is true for every x^e_α $\in FSP(X)_E$.

(3) $\Longrightarrow$ (1): Let $Fcl(x^e_\alpha) = x^e_\alpha\ \forall\ x^e_\alpha \in FSP(X)_E$ and $x^e_\alpha\ \overline{q}\ y^t_\beta$. Then $x^e_\alpha,\ y^t_\beta \in \tau^c$. Since $y^t_\beta\ \overline{q}$ $(y^t_\beta)^c = O_{x^e_\alpha}$ and $x^e_\alpha\ \overline{q}\ (x^e_\alpha)^c = O_{y^t_\beta}$. Hence, (X, τ, E) is a FST_1-space.

Theorem 4.3. Let (X, τ, E) be a fuzzy soft topological space. If (X, τ, E) is a FST_2-space, then $x^e_\alpha = \sqcap\{Fcl(O_{x^e_\alpha}); O_{x^e_\alpha} \in N(x^e_\alpha)\}$ for all $x^e_\alpha \in FSP(X)_E$.

Proof. Let (X, τ, E) be a FST_2-space and $x^e_\alpha \in FSP(X)_E$. Then for any $x^e_\alpha\ \overline{q}\ y^t_\beta$ there exist $O_{x^e_\alpha}$ and $O_{y^t_\beta}$ such that $O_{x^e_\alpha}\ \overline{q}\ O_{y^t_\beta}$. By (3) of Proposition 2.4, we have $y^t_\beta\ \overline{q}\ Fcl(O_{x^e_\alpha})$ and so $y^t_\beta\ \overline{q}$ $\sqcap\{Fcl(O_{x^e_\alpha}); O_{x^e_\alpha} \in N(x^e_\alpha)\}$. By (6) of Proposition 2.3, $\sqcap\{Fcl(O_{x^e_\alpha}); O_{x^e_\alpha} \in N(x^e_\alpha)\} \subseteq x^e_\alpha$. But $x^e_\alpha\ \widetilde{\in}$ $\sqcap\{Fcl(O_{x^e_\alpha}); O_{x^e_\alpha} \in N(x^e_\alpha)\}$. This complete the proof.

Theorem 4.4. The following implications hold:

$$FST_4 \Longrightarrow FST_3 \Longrightarrow FST_2 \Longrightarrow FST_1 \Longrightarrow FST_0.$$

Proof. (i) $FST_4 \Longrightarrow FST_3$: Let (X, τ, E) be a FST_4-space and $x^e_\alpha\ \overline{q}\ g_B$ where $g_B \in \tau^c$. Then $Fcl(x^e_\alpha) = x^e_\alpha$ implies $Fcl(x^e_\alpha)\ \overline{q}\ g_B$. Since (X, τ, E) is a FSR_3-space, then there exist $O_{Fcl(x^e_\alpha)}$ and

$O_{g_B} \in \tau$ such that $O_{Fcl(x^e_\alpha)}\ \overline{q}\ O_{g_B}$. Now put $O_{x^e_\alpha} = O_{Fcl(x^e_\alpha)}$, then $O_{x^e_\alpha}\ \overline{q}\ O_{g_B}$. Hence (X, τ, E) is a FST_3-space.

(ii) $\mathrm{FST}_3 \Longrightarrow \mathrm{FST}_2$: Let (X, τ, E) be a FST_3-space and $x^e_\alpha\ \overline{q}\ y^t_\beta$. Then $Fcl(x^e_\alpha) = x^e_\alpha$ implies $Fcl(x^e_\alpha)\ \overline{q}\ y^t_\beta$ and $Fcl(x^e_\alpha) \in \tau^c$. Since (X, τ, E) is a FSR_2-space, then there exist $O_{Fcl(x^e_\alpha)}$ and $O_{y^t_\beta} \in \tau$ such that $O_{Fcl(x^\alpha_e)}\ \overline{q}\ O_{y^t_\beta}$. Now put $O_{x^\alpha_e} = O_{Fcl(x^\alpha_e)}$, then $O_{x^\alpha_e}\ \overline{q}\ O_{y^t_\beta}$. Hence (X, τ, E) is a FST_2-space.

(iii) $\mathrm{FST}_2 \Longrightarrow \mathrm{FST}_1$ and $\mathrm{FST}_1 \Longrightarrow \mathrm{FST}_0$ are immediate.

Corollary 4.1. The following implications hold:

$$
\begin{array}{ccccccc}
\mathrm{FST}_4 \Longrightarrow & & \mathrm{FST}_3 \Longrightarrow & & \mathrm{FST}_2 \Longrightarrow \mathrm{FST}_1 \Longrightarrow \mathrm{FST}_0. \\
\Downarrow & & \Downarrow & & \Downarrow \qquad\qquad \Downarrow \\
\mathrm{FSR}_3 \wedge \mathrm{FSR}_0 & \Longrightarrow & \mathrm{FSR}_2 & \Longrightarrow & \mathrm{FSR}_1 \Longrightarrow \mathrm{FSR}_0
\end{array}
$$

Proof. (i) The implications $\mathrm{FST}_4 \Longrightarrow \mathrm{FST}_3 \Longrightarrow \mathrm{FST}_2 \Longrightarrow \mathrm{FST}_1 \Longrightarrow \mathrm{FST}_0$ and $\mathrm{FSR}_3 \wedge \mathrm{FSR}_0 \Longrightarrow \mathrm{FSR}_2 \Longrightarrow \mathrm{FSR}_1 \Longrightarrow \mathrm{FSR}_0$ are hold by Theorems 4.4, 3.2.

(ii) $\mathrm{FST}_3 \Longrightarrow \mathrm{FSR}_2$ is obvious by from definition 4.1.

(iii) $\mathrm{FST}_2 \Longrightarrow \mathrm{FSR}_1$: Let (X, τ, E) be a FST_2-space and $x^e_\alpha\ \overline{q}\ y^t_\beta$. Then there exist $O_{x^e_\alpha}$ and $O_{y^t_\beta}$ such that $O_{x^e_\alpha}\ \overline{q}\ O_{y^t_\beta}$. This implies that $y^t_\beta\ \overline{q}\ O_{x^e_\alpha}$ and $x^e_\alpha\ \overline{q}\ O_{y^t_\beta}$. By (3) of Proposition 2.4, we have $y^t_\beta\ \overline{q}\ Fcl(x^e_\alpha)$ and $x^e_\alpha\ \overline{q}\ Fcl(y^t_\beta)$. Hence (X, τ, E) is a FSR_1-space.

(iv) $\mathrm{FST}_1 \Longrightarrow \mathrm{FSR}_0$: Let (X, τ, E) be a FST_1-space. By (3) of Theorem 4.2, then $Fcl(x^e_\alpha) = x^e_\alpha$ $\forall x^e_\alpha \in FSP(X)_E$. Since $x^e_\alpha\ \tilde{\in}\ O_{x^e_\alpha}$ $\forall O_{x^e_\alpha} \in N(x^e_\alpha)$, then $Fcl(x^e_\alpha) \subseteq O_{x^e_\alpha}$ $\forall O_{x^e_\alpha} \in N(x^e_\alpha)$. By (2) of Theorem 3.1, we have (X, τ, E) be a FSR_0-space.

(v) $\mathrm{FST}_4 \Longrightarrow \mathrm{FSR}_3 \wedge \mathrm{FSR}_0$: Let (X, τ, E) be a FST_4-space. Then, (X, τ, E) is a FSR_3 and FST_1-space. Hence, (X, τ, E) is a FSR_3 and FSR_0-space.

Definition 4.2. The property P is said to be a hereditary property if (X, τ, E) is a fuzzy soft topological space has the property P, then every fuzy soft subspace has the P.

Now, the following theorems shows that the axioms FSR_i for $i = 0, 1, 2$ and FST_j for $j = 0, 1, 2, 3$ are hereditary properties.

Definition 4.3. Let $f_A \in FSS(X)_E$. The support of $f_A(e)$, denoted by $S(f_A(e))$, is the set, $S(f_A(e)) = \{x \in X; f_A(e)(x) > 0\}$.

Definition 4.4. A fuzzy soft set g_B is said to be quasi-coincident with h_C with respect to a fuzzy soft set f_A, denoted by $g_B\ q_{f_A} h_C$, if there exists $x \in S(f_A(e))$ such that $\mu^e_{g_B}(x) + \mu^e_{h_C}(x) > \mu^e_{f_A}(x)$, for some $e \in (A \cap B) \cap C$. In particular, $x^e_\alpha\ q_{f_A} g_B$ if $\alpha + \mu^e_{g_B}(x) > \mu^e_{f_A}(x)$.

Theorem 4.5. Let (f_A, τ_{f_A}, A) be a fuzzy soft topological space and g_B be a fuzzy soft subset of f_A. If (f_A, τ_{f_A}, A) is a FST_j-space, then (g_B, τ_{g_B}, B) is a FST_j-space for $j = 0, 1, 2, 3$.

Proof. As a sample, we will prove the case $j = 0$. Let (f_A, τ_{f_A}, A) be a FST_0-space, x^e_α, $y^t_\beta \in FSP(g_B)_B$ with $x^e_\alpha\ \overline{q}_{g_B}\ y^t_\beta$. Then $x^e_\alpha\ \overline{q}_{f_A}\ y^t_\beta$. Since (f_A, τ_{f_A}, A) is a FST_0-space, then there exists $O_{x^e_\alpha} \in \tau_{f_A}$ such that $O_{x^e_\alpha}\ \overline{q}_{f_A}\ y^t_\beta$ or there exists $O_{y^t_\beta} \in \tau_{f_A}$ such that $O_{y^t_\beta}\ \overline{q}_{f_A} x^e_\alpha$. Take $O^*_{x^e_\alpha} = O_{x^\alpha_e} \sqcap g_B$ or $O^*_{y^t_\beta} = O_{y^t_\beta} \sqcap g_B \in \tau_{g_B}$. Therefore, $O^*_{x^e_\alpha}\ \overline{q}_{g_B}\ y^t_\beta$ or $O^*_{y^t_\beta}\ \overline{q}_{g_B}\ x^e_\alpha$. Hence (g_B, τ_{g_B}, B) is a FST_0-space.

Theorem 4.6. Let (f_A, τ_{f_A}, A) be a fuzzy soft topological space and g_B be a fuzzy soft subset of f_A. If (f_A, τ_{f_A}, A) is a FSR$_i$-space, then (g_B, τ_{g_B}, B) is a FSR$_i$-space for $i = 0, 1, 2$.

Proof. As a sample we will prove the case $i = 2$. Let (f_A, τ_{f_A}, A) be a FSR$_2$-space, $x^e_\alpha \,\widetilde{\in}\, FSP(g_B)_B$ and h_C be a fuzzy soft closed subset of g_B with $x^\alpha_e \,\overline{q}_{g_B}\, h_C$. Then $x^\alpha_e \,\overline{q}_{g_B}\, Fcl_{\tau_{g_B}}(h_C)$. Since $Fcl_{\tau_{g_B}}(h_C) = Fcl_{\tau_{f_A}}(h_C) \sqcap g_B$, then $x^\alpha_e \,\overline{q}_{g_B}\, [Fcl_{\tau_{f_A}}(h_C) \sqcap g_B]$. This implies, $\alpha + \min\{\mu^e_{Fcl_{\tau_{f_A}}(h_C)}(x) + \mu^e_{g_B}(x)\} \leqslant \mu^e_{g_B}(x)$. Now, if $\mu^e_{g_B}(x) \neq 0$, then $\mu^e_{g_B}(x) = \mu^e_{f_A}(x)$ and so $\alpha + \mu^e_{Fcl_{\tau_{f_A}}(h_C)}(x) \leqslant \mu^e_{f_A}(x)$. Therefore, $x^\alpha_e \,\overline{q}_{f_A}\, Fcl_{\tau_{f_A}}(h_C)$. If $\mu^e_{g_B}(x) = 0$, then $\alpha = 0$ and so $\alpha + \mu^e_{Fcl_{\tau_{f_A}}(h_C)}(x) \leqslant \mu^e_{f_A}(x)$. Therefore, $x^\alpha_e \,\overline{q}_{f_A}\, Fcl_{\tau_{f_A}}(h_C)$. Since (f_A, τ_{f_A}, A) is a FSR$_2$-space, then there exist $O_{x^\alpha_e}$ and $O_{Fcl_{\tau_{f_A}}(h_C)} \in \tau_{f_A}$ such that $O_{x^\alpha_e} \,\overline{q}_{f_A}\, O_{Fcl_{\tau_{f_A}}(h_C)}$. Take $O^*_{x^e_\alpha} = O_{x^\alpha_e} \sqcap g_B$, $O^*_{h_C} = O_{Fcl_{\tau_{f_A}}(h_C)} \sqcap g_B \in \tau_{g_B}$. Hence $O^*_{x^e_\alpha} \,\overline{q}_{f_A}\, O^*_{h_C}$, and so (g_B, τ_{g_B}, B) is a FSR$_2$-space.

Theorem 4.7. Let (f_A, τ_{f_A}, A) be a fuzzy soft topological space and g_B be a fuzzy soft closed subset of f_A. If (f_A, τ_{f_A}, A) is a FSR$_3$-space, then (g_B, τ_{g_B}, B) is a FSR$_3$-space.

Proof. Let (f_A, τ_{f_A}, A) be a FSR$_3$-space, $g_B \in \tau^c_{f_A}$. Suppose h_C and s_D are fuzzy soft closed subsets of g_B. Then h_C and s_D are fuzzy soft closed subsets of f_A. Since (f_A, τ_{f_A}, A) is a FSR$_3$-space, then there exist O_{h_C} and $O_{s_D} \in \tau_{f_A}$ such that $O_{h_C} \,\overline{q}_{f_A}\, O_{s_D}$. Take $O^*_{h_C} = O_{h_C} \sqcap g_B$ and $O^*_{s_D} = O_{s_D} \sqcap g_B \in \tau_{g_B}$. Hence, $O^*_{h_C} \,\overline{q}_{g_B}\, O^*_{s_D}$ and so (g_B, τ_{g_B}, B) is a FSR$_3$-space.

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