South Asian Journal of Mathematics 2017, Vol. 7 (4): 172 ∼ 181
www.sajm-online.com ISSN 2251-1512 RESEARCH ARTICLE

On decompositions of some subsets of soft weak structures

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Abstract The main purpose of this paper, is to extend the study of soft weak structures [1, 36] defined on a set X. We further introduce the soft structure $r(swo)$ and study its basic properties in detail. Also, the relationship between the soft structure $r(swo)$ and other existing soft structures [1] have been investigated, supported by counter examples. It has been pointed out in this paper that many of these parameters studied have, in fact, applications in real world situations and therefore I believe that this is an extra justification for the work conducted in this paper.

Key Words Soft weak structures, Soft minimal spaces, $\pi(swo)$, $\sigma(swo)$, $\alpha(swo)$, $\beta(swo)$, $\rho(swo)$ and $r(swo)$
MSC 2010 54A40, 54C08

1 Introduction

Csar [16] introduced a generalized structure called generalized topology. Also, Csar [15, 17], introduced and studied generalized operators. After then, Csar [18] has introduced a new notion of structures called weak structure (briefly, WS). So that, every generalized topology [16] and every minimal structure [32] is a WS. In 2007, Arpad Szaz [14] succeed to introduce an application on the minimal spaces and generalized spaces. In [18], Csar defined some structures and operators under more general conditions, which are investigated in detail in [34].

Molodtsov [33] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. In 2011, Shabir et al. [35] initiated the study of soft topological spaces. In [23], Kandil et. al. introduced some soft operations and investigated their related properties. Kandil et al. [30] introduced the notion of soft semi separation axioms. The notion of soft ideal was initiated for the first time by Kandil et al.[26]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal. Applications to various fields were further investigated in [9, 24, 25, 27, 28, 29, 31].

Citation: A. M. Abd El-latif, On decompositions of some subsets of soft weak structures, South Asian J Math, 2017, 7(4), 172-181.

The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [20]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of b-open soft sets was initiated in [6, 19]. More recent papers have studied related topological properties of supra soft topological spaces [2, 3, 4, 5, 7, 8, 10, 11, 12, 13]. The notion of soft generalized topological space was introduced by Jyothis et al. in [22], which is generalized in [21] to notion of soft minimal spaces. In 2016, Zakari et al. [36] introduced the concept of soft weak structure $\mathcal{SW}$ over a universe X with a fixed set of parameters E, which is extended in [1].

Our aim of this paper, is to introduce the soft structure $r(swo)$ and study its basic properties in detail. Also, the relationship between the soft structure $r(swo)$ and other existing soft structures [1] have been investigated.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel. For more detail see [1, 20, 35, 36, 37].

Definition 2.1. *[35] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

1. *$\tilde{X}, \tilde{\varphi} \in \tau$, where $\tilde{\varphi}(e) = \varphi$ and $\tilde{X}(e) = X$, $\forall e \in E$,*
2. *the union of any number of soft sets in τ belongs to τ,*
3. *the intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.2. *[37] The soft set $(F, E) \in SS(X)_E$ is called a soft point in $\tilde{X}$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \varphi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.*

Definition 2.3. *[37] The soft point x_e is said to be belonging to the soft set (G, E), denoted by $x_e \tilde{\in} (G, E)$, if for the element $e \in E$, $x \in G(e)$.*

Definition 2.4. *[21] Let $\tilde{m}$ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{m}$ is called a soft minimal space if and only if $\tilde{X}, \tilde{\varphi} \in \tilde{m}$.*

Definition 2.5. *[36] Let $\mathcal{SW}$ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mathcal{SW}$ is called a soft weak structure if and only if $\tilde{\varphi} \in \mathcal{SW}$. The members of $\mathcal{SW}$ are said to be $\mathcal{SW}$-open soft sets in X. A soft set (F, E) over X is said to be a $\mathcal{SW}$-closed soft set, if its relative complement $(F, E)^{\tilde{c}}$ is $\mathcal{SW}$-open soft. We denote the set of all $\mathcal{SW}$-open soft sets over X by $swo(X)$ and the set of all $\mathcal{SW}$-closed soft sets by $swc(X)$.*

Definition 2.6. *[36] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E. Then, the $\mathcal{SW}$-soft interior of (G,E), denoted by $i_{sw}(G,E)$ is the soft union of all $\mathcal{SW}$-open soft subsets of (G,E) i.e*
$i_{sw}(G,E) = \tilde{\cup}\{(H,E) : (H,E) \; is \; \mathcal{SW} - open \; soft \; set \; and \; (H,E)\tilde{\subseteq}(G,E)\}$.

Definition 2.7. *[36] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E. Then, the $\mathcal{SW}$-soft closure of (F,E), denoted by $c_{sw}(F,E)$ is the soft intersection of all $\mathcal{SW}$-closed soft supersets of (F,E) i.e*
$c_{sw}(F,E) = \tilde{\cap}\{(H,E) : (H,E) \; is \; \mathcal{SW} - closed \; soft \; set \; and \; (F,E)\tilde{\subseteq}(H,E)\}$.

Theorem 2.8. *[36] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E),(G,E) \in SS(X)_E$. Then, the following statements hold.*

1. $i_{sw}(F,E)\tilde{\subseteq}(F,E)\tilde{\subseteq}c_{sw}(F,E)$.
2. $c_{sw}(c_{sw}(F,E)) = c_{sw}(F,E)$ *and* $i_{sw}(i_{sw}(F,E)) = i_{sw}(F,E)$.
3. *If* $(F,E)\tilde{\subseteq}(G,E)$, *then* $i_{sw}(F,E)\tilde{\subseteq}i_{sw}(G,E)$ *and* $c_{sw}(F,E)\tilde{\subseteq}c_{sw}(G,E)$.
4. $c_{sw}(\tilde{X} - (F,E)) = \tilde{X} - i_{sw}(F,E)$ *and* $i_{sw}(\tilde{X} - (F,E)) = \tilde{X} - c_{sw}(F,E)$.

Theorem 2.9. *[1] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E),(G,E) \in SS(X)_E$. Then, the following statements hold.*

1. $x_e\tilde{\in}c_{sw}(F,E)$ *if and only if* $(G,E)\tilde{\cap}(F,E) \neq \tilde{\varphi}$ *for every $\mathcal{SW}$-open soft set* (G,E) *and* $x_e\tilde{\in}(G,E)$.
2. $x_e\tilde{\in}i_{sw}(F,E)$ *if and only if there exists a $\mathcal{SW}$-open soft set (G,E) such that* $x_e\tilde{\in}(G,E)$ $\tilde{\subseteq}(F,E)$.

Theorem 2.10. *[1] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E),(G,E) \in SS(X)_E$. Then:*

1. $i_{sw}((F,E)\tilde{\cap}(G,E))\tilde{\subseteq}i_{sw}(F,E)\tilde{\cap}i_{sw}(G,E)$.
2. $c_{sw}(F,E)\tilde{\cup}c_{sw}(G,E)\tilde{\subseteq}c_{sw}((F,E)\tilde{\cup}(G,E))$.
3. $d_{sw}(F,E)\tilde{\cup}d_{sw}(G,E)\tilde{\subseteq}d_{sw}((F,E)\tilde{\cup}(G,E))$.

Definition 2.11. *[36] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E) \in SS(X)_E$. Then, (F,E) is called $\mathcal{SW}$-soft dense if and only if $c_{sw}(F,E) = \tilde{X}$.*

Theorem 2.12. *[1] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E) \in SS(X)_E$. Then:*

1. $i_{sw}(c_{sw}(i_{sw}(c_{sw}(F,E)))) = i_{sw}(c_{sw}(F,E))$.
2. $c_{sw}(i_{sw}(c_{sw}(i_{sw}(F,E)))) = c_{sw}(i_{sw}(F,E))$.

Definition 2.13. *[1] Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E) \in SS(X)_E$. Then:*

1. $(F,E) \in \pi(swo)$ *if and only if* $(F,E)\tilde{\subseteq} i_{sw}(c_{sw}(F,E))$ *and* $(F,E) \in \pi(swc)$ *if and only if* $c_{sw}(i_{sw}(F,E))\tilde{\subseteq}(F,E)$.

2. $(F,E) \in \sigma(swo)$ *if and only if* $(F,E)\tilde{\subseteq} c_{sw}(i_{sw}(F,E))$ *and* $(F,E) \in \sigma(swc)$ *if and only if* $i_{sw}(c_{sw}(F,E))\tilde{\subseteq}(F,E)$.

3. $(F,E) \in \alpha(swo)$ *if and only if* $(F,E)\tilde{\subseteq} i_{sw}(c_{sw}(i_{sw}(F,E)))$ *and* $(F,E) \in \alpha(swc)$ *if and only if* $c_{sw}(i_{sw}(c_{sw}(F,E)))\tilde{\subseteq}(F,E)$.

4. $(F,E) \in \beta(swo)$ *if and only if* $(F,E)\tilde{\subseteq} c_{sw}(i_{sw}(c_{sw}(F,E)))$ *and* $(F,E) \in \beta(swc)$ *if and only if* $i_{sw}(c_{sw}(i_{sw}(F,E)))\tilde{\subseteq}(F,E)$.

5. $(F,E) \in \rho(swo)$ *if and only if* $(F,E)\tilde{\subseteq} i_{sw}(c_{sw}(F,E))\tilde{\cup} c_{sw}(i_{sw}(F,E))$ *and* $(F,E) \in \rho(swc)$ *if and only if* $i_{sw}(c_{sw}(F,E))\tilde{\cap} c_{sw}(i_{sw}(F,E))\tilde{\subseteq}(F,E)$.

3 Characterizations of The Soft Structure $r(swo)$

In this section, we introduce and study properties of the soft structure $r(swo)$ in detail. Furthermore, the relationship between the soft structure $r(swo)$ and other existing soft structures [1] have been investigated.

Definition 3.1. *Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E and $(F,E) \in SS(X)_E$. Then, $(F,E) \in r(swo)$ if and only if $(F,E) = i_{sw}(c_{sw}(F,E))$ and $(F,E) \in r(swc)$ if and only if $(F,E) = c_{sw}(i_{sw}(F,E))$.*

Theorem 3.2. *Let $\mathcal{SW}$ be any soft weak structure on X and $(F,E) \in SS(X)_E$. Then, $(F,E) \in r(swo)$ if and only if $(F,E) \in \alpha(swo)$ and $(F,E)^{\tilde{c}} \in \beta(swo)$.*

Proof. **Necessity:** Let $(F,E) \in r(swo)$. Then, $(F,E) = i_{sw}(c_{sw}(F,E))$ from Definition 2.13. Hence, $i_{sw}(F,E) = i_{sw}(i_{sw}(c_{sw}(F,E))) = i_{sw}(c_{sw}(F,E)) = (F,E)$ from Theorem 2.8 (2). Hence, $(F,E) = i_{sw}(F,E) = i_{sw}(i_{sw}(F,E))\tilde{\subseteq} i_{sw}(c_{sw}(i_{sw}(F,E)))$. Thus, $(F,E) \in \alpha(swo)$. On the other hand, $(F,E)^{\tilde{c}} = [i_{sw}(c_{sw}(F,E))]^{\tilde{c}} = c_{sw}(i_{sw}(F,E)^{\tilde{c}})\tilde{\subseteq} c_{sw}(i_{sw}(c_{sw}(F,E)^{\tilde{c}}))$. Thus, $(F,E)^{\tilde{c}} \in \beta(swo)$.
Sufficient: Let $(F,E) \in \alpha(swo)$ and $(F,E)^{\tilde{c}} \in \beta(swo)$ for any $(F,E) \in SS(X)_E$. Then, $(F,E) \in \beta(swc)$. It follows, $i_{sw}(c_{sw}(i_{sw}(F,E)))\tilde{\subseteq}(F,E)\tilde{\subseteq} i_{sw}(c_{sw}(i_{sw}(F,E)))$, and hence $(F,E) = i_{sw}(c_{sw}(i_{sw}(F,E)))$. Thus, $i_{sw}(c_{sw}(F,E)) = i_{sw}(c_{sw}(i_{sw}(c_{sw}(i_{sw}(F,E))))) = i_{sw}(c_{sw}(i_{sw}(F,E))) = (F,E)$ from Theorem 2.12 (2). Therefore, $(F,E) \in r(swo)$. □

Theorem 3.3. *Let $\mathcal{SW}$ be any soft weak structure on X and $(F,E) \in SS(X)_E$. Then, $(F,E) \in r(swo)$ if and only if $(F,E) \in \pi(swo)$ and $(F,E)^{\tilde{c}} \in \sigma(swo)$.*

Proof. **Necessity:** Follows directly from Definition 2.13.

Sufficient: Let $(F,E) \in \pi(swo)$ and $(F,E)^{\tilde{c}} \in \sigma(swo)$ for any $(F,E) \in SS(X)_E$. Then, $(F,E) \in \sigma(swc)$. It follows, $i_{sw}(c_{sw}(F,E))\tilde{\subseteq}(F,E)\tilde{\subseteq}i_{sw}(c_{sw}(F,E))$, and hence $(F,E) = i_{sw}(c_{sw}(F,E))$. Thus, $(F,E) \in r(swo)$. □

Theorem 3.4. *Let $\mathcal{SW}$ be any soft weak structure on X and $(F,E) \in SS(X)_E$. Then, $(F,E) \in \pi(swo)$ if and only if there exists $(G,E) \in r(swo)$ such that $(F,E)\tilde{\subseteq}(G,E)$ and $c_{sw}(F,E) = c_{sw}(G,E)$.*

Proof. **Necessity:** Let $(F,E) \in \pi(swo)$. Then, $(F,E)\tilde{\subseteq}i_{sw}(c_{sw}(F,E))$. Put $(G,E) = i_{sw}(c_{sw}(F,E))$, then $(F,E)\tilde{\subseteq}(G,E)$ and $c_{sw}(F,E)\tilde{\subseteq}c_{sw}(G,E)$ from Theorem 2.8 (3). But, $c_{sw}(G,E) = c_{sw}(i_{sw}(c_{sw}(F,E)))$ $\tilde{\subseteq}c_{sw}(F,E)$. Thus, $c_{sw}(F,E) = c_{sw}(G,E)$. Also, we have $i_{sw}(c_{sw}(G,E)) = i_{sw}(c_{sw}(i_{sw}(c_{sw}(F,E)))) = i_{sw}(c_{sw}(F,E)) = (G,E)$ from Theorem 2.12 (1). Therefore, $(G,E) \in r(swo)$.

Sufficient: Let $(F,E)\tilde{\subseteq}(G,E)$ and $c_{sw}(F,E) = c_{sw}(G,E)$ for some $(G,E) \in r(swo)$, it follows that $i_{sw}(c_{sw}(F,E)) = i_{sw}(c_{sw}(G,E)) = (G,E)$. Hence, $(F,E)\tilde{\subseteq}(G,E) = i_{sw}(c_{sw}(F,E))$, and hence $(F,E) \in \pi(swo)$. □

Theorem 3.5. *Let $\mathcal{SW}$ be any soft weak structure on X and $(F,E) \in SS(X)_E$. If $(F,E) \in \pi(swo)$, then $(F,E) = (G,E)\tilde{\cap}(H,E)$ for some $(G,E) \in r(swo)$ and (H,E) is $\mathcal{SW}$-soft dense set.*

Proof. Let $(F,E) \in \pi(swo)$. By Theorem 3.4, there exists $(G,E) \in r(swo)$ such that $(F,E)\tilde{\subseteq}(G,E)$ and $c_{sw}(F,E) = c_{sw}(G,E)$. Take $(H,E) = (F,E)\tilde{\cup}(G,E)^{\tilde{c}}$. Then, $\tilde{X} = (G,E)\tilde{\cup}(G,E)^{\tilde{c}}\tilde{\subseteq}c_{sw}(G,E)\tilde{\cup}c_{sw}((G,E)^{\tilde{c}}) = c_{sw}(F,E)\tilde{\cup}c_{sw}((G,E)^{\tilde{c}})\tilde{\subseteq}c_{sw}[(F,E)\tilde{\cup}(G,E)^{\tilde{c}}] = c_{sw}(H,E)$ from Theorem 2.10 (2). But, $c_{sw}(H,E)\tilde{\subseteq}\tilde{X}$. It follows that, $c_{sw}(H,E) = \tilde{X}$. Thus, (H,E) is $\mathcal{SW}$-soft dense set. Moreover, $(F,E) = (G,E)\tilde{\cap}(H,E)$. □

Remark 3.6. *The converse of Theorem 3.5 is not true in general as shown in the following example.*

Example 3.7. *Suppose that there are three alternatives in the universe of cars $X = \{c_1, c_2, c_3\}$ and consider $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "Motor" and "color" respectively. Let $\mathcal{SW} = \{\tilde{\varphi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$, where $(F_1,E), (F_2,E), (F_3,E)$,*
(F_4,E) are four soft sets over X representing the attractiveness of the cars which Mr. A and Mr. B are going to buy defined as follows:
$F_1(e_1) = \{c_1\}, \quad F_1(e_2) = \{c_2\}$,
$F_2(e_1) = \{c_1, c_2\}, \quad F_2(e_2) = \{c_1, c_2\}$,
$F_3(e_1) = \{c_2, c_3\}, \quad F_3(e_2) = \{c_1, c_3\}$,
$F_4(e_1) = \{c_1, c_3\}, \quad F_4(e_2) = \{c_1, c_2\}$.
Consider the soft sets $(M,E), (N,E)$, where:
$M(e_1) = \{c_2, c_3\}, \quad M(e_2) = \{c_1, c_3\}$,
$N(e_1) = \{c_1, c_2\}, \quad M(e_2) = \{c_2, c_3\}$,
we have $(M,E) \in r(swo)$ and (M,E) is $\mathcal{SW}$-soft dense set. But, $(M,E)\tilde{\cap}(N,E) = (O,E) \notin \pi(swo)$, where:
$O(e_1) = \{c_2\}, \quad O(e_2) = \{c_3\}$.

Theorem 3.8. *Let $\mathcal{SW}$ be a soft weak structure on X and $(F,E) \in SS(X)_E$. Then, the following properties are equivalent:*

1. $(F, E) \in \beta(swo)$,

2. *There exists* $(G, E) \in \pi(swo)$ *such that* $(G, E)\tilde{\subseteq}c_{sw}(F, E)\tilde{\subseteq}c_{sw}(G, E)$,

3. $c_{sw}(F, E) \in r(swc)$.

Proof. (1) $\Rightarrow$ (2). Let $(F, E) \in \beta(swo)$. Then, $(F, E)\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(F, E)))$. Put $(G, E) = i_{sw}(c_{sw}(F, E))$. It follows that, $(F, E)\tilde{\subseteq}c_{sw}(G, E)$, $(G, E)\tilde{\subseteq}c_{sw}(F, E)\tilde{\subseteq}c_{sw}(G, E)$ and $(G, E) = i_{sw}(c_{sw}(F, E))\tilde{\subseteq}i_{sw}(c_{sw}(G, E))$ from Theorem 2.8 (3). Hence, $(G, E) \in \pi(swo)$.

(2) $\Rightarrow$ (3). Let $(G, E) \in \pi(swo)$ such that $(G, E)\tilde{\subseteq}c_{sw}(F, E)\tilde{\subseteq}c_{sw}(G, E)$. Then, $(G, E)\tilde{\subseteq}i_{sw}(c_{sw}(G, E))$ and $c_{sw}(G, E)\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(G, E)))\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(F, E)))\tilde{\subseteq}c_{sw}(G, E)$ from (2) and Theorem 2.8 (2). So, $c_{sw}(i_{sw}(c_{sw}(F, E))) = c_{sw}(G, E)$. Since $(G, E)\tilde{\subseteq}c_{sw}(F, E)$, $c_{sw}(G, E)\tilde{\subseteq}c_{sw}(c_{sw}(F, E)) = c_{sw}(F, E)\tilde{\subseteq}c_{sw}(G, E)$ from Theorem 2.8 (2). This means that, $c_{sw}(G, E) = c_{sw}(F, E)$, and hence $c_{sw}(i_{sw}(c_{sw}(F, E))) = c_{sw}(G, E) = c_{sw}(F, E)$. Thus, $c_{sw}(F, E) \in r(swc)$.

(3) $\Rightarrow$ (1). Let $c_{sw}(F, E) \in r(swc)$. Then, $(F, E)\tilde{\subseteq}c_{sw}(F, E) = c_{sw}(i_{sw}(c_{sw}(F, E)))$. Hence, $(F, E) \in \beta(swo)$.

□

Theorem 3.9. *Let $\mathcal{SW}$ be a soft weak structure on X. If $(F, E)\tilde{\subseteq}(G, E)\tilde{\subseteq}c_{sw}(F, E)$ and $(F, E) \in \beta(swo)$, then $(G, E) \in \beta(swo)$.*

Proof. Let $(F, E)\tilde{\subseteq}(G, E)\tilde{\subseteq}c_{sw}(F, E)$ and $(F, E) \in \beta(swo)$. Then, $(F, E)\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(F, E)))$. Since $(G, E)\tilde{\subseteq}c_{sw}(F, E)$, $(G, E)\tilde{\subseteq}c_{sw}(F, E)\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(F, E)))\tilde{\subseteq}c_{sw}(i_{sw}(c_{sw}(G, E)))$ from Theorem 2.8 (3). Therefore, $(G, E) \in \beta(swo)$. □

Definition 3.10. *[1] A soft set (F, E) in a soft weak structure $\mathcal{SW}$ is said to be a $\mathcal{SW}$-clopen soft if and only if it is both $\mathcal{SW}$-open soft and $\mathcal{SW}$-closed soft.*

Theorem 3.11. *[1] Let $\mathcal{SW}$ be any soft weak structure on X and $(F, E) \in SS(X)_E$. If (F, E) is $\mathcal{SW}$-clopen soft set, then $(F, E) \in \alpha(swo)$ and $(F, E)^{\tilde{c}} \in \pi(swo)$.*

Theorem 3.12. *Let $\mathcal{SW}$ be a soft weak structure on X and $(F, E), (G, E) \in SS(X)_E$. Then, the following properties are hold:*

1. *If (F, E) is $\mathcal{SW}$-clopen soft, then $(F, E) \in r(swo)$ and $(F, E) \in r(swc)$.*

2. *$(F, E) \in \beta(swo)$ if and only if $c_{sw}(F, E) = c_{sw}(i_{sw}(c_{sw}(F, E)))$ if and only if $c_{sw}(F, E) \in \beta(swo)$.*

3. *$(F, E) \in \sigma(swo)$ if and only if $c_{sw}(F, E) = c_{sw}(i_{sw}(F, E))$.*

4. *If (F, E) is $\mathcal{SW}$-clopen soft, then $(F, E) \in \alpha(swo)$ and $(F, E)^{\tilde{c}} \in \alpha(swo)$.*

5. *$(F, E) \in r(swo)$ if and only if $(F, E)^{\tilde{c}} \in r(swo)$.*

Proof. 1. Follows from the fact that, if (F,E) is $\mathcal{SW}$-open (resp. closed) soft, then $i_{sw}(F,E) = (F,E)$ (resp. $c_{sw}(F,E) = (F,E)$).

2. Follows directly from Definition 2.13 (4).

3. $(F,E) \in \sigma(swo)$ if and only if $(F,E)\tilde{\subseteq}c_{sw}(i_{sw}(F,E))$ if and only if $c_{sw}(F,E)\tilde{\subseteq}c_{sw}(i_{sw}(F,E))\tilde{\subseteq}$ $c_{sw}(F,E)$ if and only if $c_{sw}(F,E) = c_{sw}(i_{sw}(F,E))$.

4. It is similar to the proof of (1).

5. Clear from Definition 3.1.

□

Remark 3.13. *Theorem 3.12 (4) is a a generalization of Theorem 3.11. Also, the following example shows that, the converse of Theorem 3.12 (4) is not true in general.*

Example 3.14. *Suppose that there are four alternatives in the universe of phones* $X = \{p_1, p_2, p_3, p_4\}$ *and consider* $E = \{e_1, e_2\}$ *be the set of decision parameters which are stands for "android" and "expensive" respectively. Let* $\mathcal{SW} = \{\tilde{\varphi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E),$ $(F_6,E)\}$*, where* $(F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E)$ *are six soft sets over* X *representing the attractiveness of the phones which Mr.* X *and Mr.* Y *are going to buy defined as follows:*

$F_1(e_1) = \{p_4\}, \quad F_1(e_2) = \varphi,$
$F_2(e_1) = \{p_1\}, \quad F_2(e_2) = \{p_3\},$
$F_3(e_1) = \{p_1, p_2\}, \quad F_3(e_2) = \{p_1, p_2\},$
$F_4(e_1) = \{p_4\}, \quad F_4(e_2) = \{p_4\},$
$F_5(e_1) = \{p_1, p_2\}, \quad F_5(e_2) = \varphi,$
$F_6(e_1) = \{p_2, p_3\}, \quad F_6(e_2) = \{p_2, p_3\}.$

Then, the soft set $(G,E) \in \alpha(swo)$ *and* $(G,E)^{\tilde{c}} \in \alpha(swo)$*. But,* $(G,E) \notin \mathcal{SW}$*, where:* $G(e_1) = \{p_1, p_2, p_3\}, \quad M(e_2) = X.$

Remark 3.15. *If* $(F,E) \in \sigma(swo)$*, then it doesn't imply that* $i_{sw}(F,E) \neq \varphi$*. The following example supports our claim.*

Example 3.16. *Suppose that there are four alternatives in the universe of jobs* $X = \{j_1, j_2, j_3, j_4\}$ *and consider* $E = \{e_1, e_2\}$ *be the set of decision parameters which are stands for "salary" and "position" respectively. Let* $\mathcal{SW} = \{\tilde{\varphi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E)\}$*, where* $(F_1,E), (F_2,E), (F_3,E), (F_4,E),$ (F_5,E) *are five soft sets over* X *representing the attractiveness of the jobs which Mr.* X *and Mr.* Y *are going to buy defined as follows:*

$F_1(e_1) = \{j_1, j_2\}, \quad F_1(e_2) = \{j_1, j_2\},$
$F_2(e_1) = \{j_1\}, \quad F_2(e_2) = \{j_3\},$
$F_3(e_1) = \{j_1, j_2\}, \quad F_3(e_2) = \varphi,$
$F_4(e_1) = \{j_1, j_2, j_3\}, \quad F_4(e_2) = \{j_1, j_2, j_3\},$
$F_5(e_1) = \{j_2, j_3\}, \quad F_5(e_2) = \{j_2, j_3\}.$

Then, the soft set $(T, E) \in \sigma(swo)$, but $i_{sw}(T, E) = \varphi$, where:
$T(e_1) = \{j_4\}, \quad T(e_2) = \varphi$.

Theorem 3.17. *If $X \in \mathcal{SW}$. Then, $i_{sw}(F, E) \neq \varphi$ for every $(F, E) \in \sigma(swo)$. The following example supports our claim.*

Proof. Let $\varphi \neq (F, E) \in \sigma(swo)$. By Definition 2.13 (2), $(F, E) \tilde{\subseteq} c_{sw}(i_{sw}(F, E))$. If $i_{sw}(F, E) = \varphi$, then $c_{sw}(i_{sw}(F, E)) = \varphi$, where $X \in \mathcal{SW}$. Thus, $(F, E) = \varphi$, which is a contradiction. Therefore, $i_{sw}(F, E) = \varphi$. □

Remark 3.18. *Examples 3.9 of [1], show that the equality will not hold in Theorem 2.10. The following theorem shows that equality holds, if $\mathcal{SW}$ is closed under finite soft intersection.*

Theorem 3.19. *Let $\mathcal{SW}$ be a soft weak structure over X with a fixed set of parameters E such that $\mathcal{SW}$ is closed under finite soft intersection and $(F, E), (G, E) \in SS(X)_E$. Then, the the following statements hold.*

1. $c_{sw}(F, E) \tilde{\cup} c_{sw}(G, E) = c_{sw}((F, E) \tilde{\cup} (G, E))$.
2. $i_{sw}((F, E) \tilde{\cap} (G, E)) = i_{sw}(F, E) \tilde{\cap} i_{sw}(G, E)$.
3. $d_{sw}(F, E) \tilde{\cup} d_{sw}(G, E) = d_{sw}((F, E) \tilde{\cup} (G, E))$.
4. $(G, E) \tilde{\cap} c_{sw}(F, E) \tilde{\subseteq} c_{sw}[(F, E) \tilde{\cap} (G, E)]$ *for every* $(G, E) \in \mathcal{SW}$.
5. $c_{sw}[(G, E) \tilde{\cap} c_{sw}(F, E)] = c_{sw}[(G, E) \tilde{\cap} (F, E)]$ *for every* $(G, E) \in \mathcal{SW}$.
6. $c_{sw}(G, E) = c_{sw}[(G, E) \tilde{\cap} (F, E)]$ *for every* $(G, E) \in \mathcal{SW}$ *and* (F, E) *is* $\mathcal{SW}$*-soft dense set.*

Proof. 1. By Theorem 2.10 (2), $c_{sw}(F, E) \tilde{\cup} c_{sw}(G, E) \tilde{\subseteq} c_{sw}((F, E) \tilde{\cup} (G, E))$. For the other inclusion, assume that $x_e \tilde{\notin} c_{sw}(F, E) \tilde{\cup} c_{sw}(G, E)$. It follows that, $x_e \tilde{\notin} c_{sw}(F, E)$ and $x_e \tilde{\notin} c_{sw}(G, E)$. By Theorem 2.9 (1), there exist $\mathcal{SW}$-open soft sets $(A, E), (B, E)$ containing x_e such that $(G, E) \tilde{\cap} (A, E) = \tilde{\varphi}$ and $(F, E) \tilde{\cap} (B, E) = \tilde{\varphi}$. Now, $x_e \tilde{\in} (A, E) \tilde{\cap} (B, E) \in \mathcal{SW}$. Hence, $[(A, E) \tilde{\cap} (B, E)] \tilde{\cap} [(F, E) \tilde{\cup} (G, E)] = [((A, E) \tilde{\cap} (B, E)) \tilde{\cap} (F, E)] \tilde{\cup} [((A, E) \tilde{\cap} (B, E)) \tilde{\cap} (G, E)] \tilde{\subseteq} [(B, E) \tilde{\cap} (F, E)] \tilde{\cup} [(A, E) \tilde{\cap} (G, E)] = \tilde{\varphi}$. Thus, $x_e \tilde{\notin} c_{sw}((F, E) \tilde{\cup} (G, E))$. Therefore, $c_{sw}(F, E) \tilde{\cup} c_{sw}(G, E) \tilde{\supseteq} c_{sw}((F, E) \tilde{\cup} (G, E))$ and so
$c_{sw}(F, E) \tilde{\cup} c_{sw}(G, E) = c_{sw}((F, E) \tilde{\cup} (G, E))$.

2. Follows from (1).

3. By a similar argument of (1).

4. Let $x_e \tilde{\in} (G, E) \tilde{\cap} c_{sw}(F, E)$ and $(G, E) \in \mathcal{SW}$. Then, $x_e \tilde{\in} (G, E)$ and $x_e \tilde{\in} c_{sw}(F, E)$. By Theorem 2.9 (1), $(F, E) \tilde{\cap} (H, E) \neq \tilde{\varphi}$ for every $\mathcal{SW}$-open soft set (H, E) containing x_e. Since $x_e \tilde{\in} (G, E) \tilde{\cap} (H, E) \in \mathcal{SW}$ and so $((G, E) \tilde{\cap} (H, E)) \tilde{\cap} (F, E) = (H, E) \tilde{\cap} [(G, E) \tilde{\cap} (F, E)]$
$\neq \tilde{\varphi}$. Hence, $x_e \tilde{\in} c_{sw}[(F, E) \tilde{\cap} (G, E)]$. Therefore, $(G, E) \tilde{\cap} c_{sw}(F, E) \tilde{\subseteq} c_{sw}[(F, E) \tilde{\cap} (G, E)]$.

5. From (4), $(G,E)\tilde{\cap}c_{sw}(F,E)\tilde{\subseteq}c_{sw}[(F,E)\tilde{\cap}(G,E)]$ for every $(G,E)\in\mathcal{SW}$, and hence $c_{sw}[(G,E)\tilde{\cap}c_{sw}(F,E)]\tilde{\subseteq}c_{sw}[(F,E)\tilde{\cap}(G,E)]$ from Theorem 2.8 (3). On the other hand, $(G,E)\tilde{\cap}(F,E)\tilde{\subseteq}(G,E)\tilde{\cap}c_{sw}(F,E)\tilde{\subseteq}c_{sw}[(G,E)\tilde{\cap}c_{sw}(F,E)]$ and so $c_{sw}[(G,E)\tilde{\cap}(F,E)]\tilde{\subseteq}c_{sw}[(G,E)\tilde{\cap}c_{sw}(F,E)]$ from Theorem 2.8 (3). Therefore, $c_{sw}[(G,E)\tilde{\cap}c_{sw}(F,E)]=c_{sw}[(G,E)\tilde{\cap}(F,E)]$.

6. Follows from (5).

□

4 Conclusion

The concept of of soft weak structures was first introduced by Zakari et al.[36] in 2016 as a generalization to soft topological spaces [35], supra soft topological spaces [20] and soft minimal spaces [21]. In [1], the deviations between soft weak structure and that in soft topological spaces and supra soft topological spaces have been investigated by Abd El-laif. Also, new soft structures, as a generalizations to thats in [18, 20, 23, 34] are introduced and their properties are investigated.

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