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Soft idealization of some generalized soft open sets

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Abstract This paper aims to introduce soft pre-open, soft α-open and soft β-open sets modulo soft ideals in soft topological spaces as a generalization of the usual notion of soft pre-openness, soft α-openness, soft β-openness, soft pre I-openness, soft αI-openness and soft β I-openness. Various fundamental properties related to these concepts with illustrative Examples will be studied.

Key Words Soft set; Soft topology; Soft bitopological spaces; pairwise soft continuous; $p\lambda$-soft continuous
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1 Introduction

Shabir and Naz [15] employed the concept of soft sets [10] to introduce idea of soft topological spaces. Since then, a number of related definitions have been conceptualized. Chen [5] introduced soft semi open sets and related properties. Arockiarani et al. [4] defined soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Akdag [1] introduced soft α-sets on soft topological spaces and study some of their properties. The interest in the soft idealized version of many soft topological properties has grown drastically in the recent years. This paper aims to introduce soft pre-open, soft α-open and soft β-open sets modulo soft ideals in soft topological spaces as a generalization of the usual notion of soft pre-openness, soft α-openness, soft β-openness, soft pre I-openness, soft α I-openness and soft β I-openness considered in [1, 2, 4, 5, 7]. Various fundamental properties related to these concepts with illustrative Examples will be studied

2 Preliminaries

In this manuscript we will benefit from many of definitions, properties and results that reported in [1-8,10, 12, 15-17].

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Definition 2.1[12] A soft topological space (X, τ, A) is called (i) Soft hyper connected, if every non-empty soft open set over set X is soft dense, (ii) Soft locally indiscrete, if every soft open set over set X is soft closed, (iii) Soft extremally disconnected, if the soft closure of every soft open set of X is soft open.

Definition 2.2[6] A soft ideal I over X is a collection of soft sets over X which satis?es the following properties: (i) If (F, A)∈I and (H, A)⊑(F, A), then (H, A)∈I. (ii) If (F, A)∈I and (H, A)∈I, then (F, A)⊔(H, A)∈I.

Definition 2.3[12] Let I be a soft ideal on a soft topological space (X, τ, A). A soft set (F, A) in is called a soft semi-open modulo a soft ideal I (written as, soft I semi-open), if there exist a soft open set (H, A) such that [(H, A)-(F, A)]∈I and [(F, A)-Scl(H, A)]∈I. A soft set (F, A) is called soft I semi-closed, if its complement $(F, A)^c$ is soft I semi-open. In other words, a soft set (F, A) is called soft I semi-closed, if there exist a soft closed set (K, A) such that (Sint(K, A)-(F, A))∈I and ((F, A)-(K, A))∈I. It is easy to see that $\emptyset_A$ and X_A are always soft I semi-open and soft I semi-closed sets.

3 Main Results

Definition 3.1 Let I be a soft ideal on a soft topological space (X, τ, A), then a soft set (F, A) is called a soft pre-open modulo soft ideal I (written as, soft I pre-open), if there exist a soft open set (H, A) such that [(F, A)-(H, A)]∈I and [(H, A)-Scl(F, A)]∈I. A soft set (F, A) is called soft I pre-closed if its soft complement is a soft I pre-open. In other words, a soft set (F, A) is called soft I pre-closed, if there exist a soft closed set (K, A) such that [Sint(F, A)-(K, A)]∈I and [(K, A)-(F, A)]∈I. A soft set (F, A) is said to be soft I pre-clopen, if it is both soft I pre-open and I pre-closed.

Definition 3.2 Let I be a soft ideal on a soft topological space (X, τ, A), then a soft set (F, A) over X is said to be soft α-open with respect to soft ideal I (written as soft I α-open), if there exist a soft open set (H, A) such that [(F, A)-Sint(Scl(H, A)]∈I and [(H, A)-(F, A)]∈I. A soft set (F, A) is soft I α-closed if its soft complement is soft I α-open set. In other words, a soft set (F, A) is said to be soft I α-closed, if there exist a soft closed set (K, A) such that [Scl(Sint(K, A))-(F, A)]∈I and [(F, A)-(K, A)]∈I. A soft set (F, A) is said to be soft I α-clopen, if it is both soft I α-open and I α-closed.

Definition 3.3 Let I be a soft ideal on a soft topological space (X, τ, A), then a soft set (F, A) is said to be soft β-open with respect to an ideal I (written as soft I β-open), if there exist a soft open set (H, A) such that [(F, A)-Scl(H, A)]∈I and [(H, A)-Scl(F, A)]∈I. A soft set (F, A) is soft I β-closed set if its soft complement is soft I β-open set. A soft set (F, A) is said to be soft I -clopen, if it is both soft I β-open and I -closed.

Example 3.1 $\emptyset_A$ and X_A are always soft I pre-(resp., α-, β-) clopen sets.

Remark 3.1 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A)∈I, then it is soft I pre-(resp., α-, β-) open.

Lemma 3.1 Let I be a soft ideal on a soft topological space (X, τ, A), then the following statements hold:

(i) Every soft pre-open set is soft I pre-open,

(ii) Every soft α-open set is soft I α-open,

(iii) Every soft β-open set is soft I β-open.

Proof. we prove only (i) and the rest of the **Proof.** is similar. (i) Let (F, A) be a soft pre-open set, then (F, A)$\sqsubseteq$Sint Scl(F, A). Suppose (H, A)=Sint Scl(F, A), then (F, A)$\sqsubseteq$ (H, A) and (H, A)$\sqsubseteq$Scl(F, A). Hence, there exist soft open set (H, A) such that [(F, A)-(H, A)]=$\emptyset_A$$\in$I and [(H,A)-Scl(F, A)]= $\emptyset_A$$\in$I and so (F, A) is soft I pre-open set.

The next Examples will explain that the converse of Lemma 3.1 cannot be achieved.

Example 3.2 Let X=A=, τ={$\emptyset_A$, X_A}$\sqcup${(Hi, A) Hi(n)={1, 2, ..., i} for every i, n} and I= SS(X, A), then (i) [10] the soft set (F, A) with F(n)={2, 4, 6, ...}, for every $n \in N$, is soft I α-open set and it is not soft α-open. (ii) the soft set (F, A) with F(n)={3, 5}, for every n, is soft I pre-open set and it is not soft pre-open.

Example 3.3 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A), (C_7, A)} where (F_1, A), (F_2, A), (F_3, A), (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A) and (C_7, A) are soft sets over X, defined as follow (F_1, A)(e_1)={a, b}, (F_1, A)(e_2)={b, c}, (F_2, A)(e_1)={b, c}, (F_2, A)(e_2)={a}, (F_3, A)(e_1)={b}, (F_3, A)(e_2)= $\emptyset$, (C_1, A)(e_1)={c}, (C_1, A)(e_2)={a}, (C_2, A)(e_1)={b}, (C_2, A)(e_2)=$\emptyset$, (C_3, A)(e_1)={c}, (C_3, A)(e_2)=$\emptyset$, (C_4, A)(e_1)=$\emptyset$, (C_4, A)(e_2)={a}, (C_5, A)(e_1)={b, c}, (C_5, A)(e_2)={a}, (C_6, A)(e_1)={b, c}, (C_6, A)(e_2)=$\emptyset$, (C_7, A)(e_1)={b}, (C_7, A)(e_2)={a}, (i) A soft set (G, A), which is de?ned as follow (G, A)(e_1)={b, c}, (G, A)(e_2)={a, b} is soft pre-open, but it is not soft open. (ii) A soft set (K, A), which is de?ned as follow (K, A)(e_1)={a, c}, (K, A)(e_2)=X is a soft I pre-open and it is not soft pre-open.

Example 3.4 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A), (C_7, A)} where (F_1, A), (F_2, A), (F_3, A), (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A), and (C_7, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a, b}, (F_1, A)(e_2)={b, c}, (F_2, A)(e_1)={b, c}, (F_2, A)(e_2)={a}, (F_3, A)(e_1)={b}, (F_3, A)(e_2)=$\emptyset$, (C_1, A)(e_1)={b}, (C_1, A)(e_2)={b, c}, (C_2, A)(e_1)={b}, (C_2, A)(e_2)={b}, (C_3, A)(e_1)={b}, (C_3, A)(e_2)={c}, (C_4, A)(e_1)={b}, (C_4, A)(e_2)=$\emptyset$, (C_5, A)(e_1)=$\emptyset$, (C_5, A)(e_2)={b, c}, (C_6, A)(e_1)=$\emptyset$, (C_6, A)(e_2)={b}, (C_7, A)(e_1)=$\emptyset$, (C_7, A)(e_2)={c}, Clearly, a soft set (G, A), which is de?ned as (G, A)(e_1)={a} (G, A)(e_2)={a}, is soft I α-open and it is not soft α-open.

Example 3.5 Let X={a, b}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (F_3, A)} are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={a}, (F_2, A)(e_1)=X, (F_2, A)(e_2)={b}, (F_3, A)(e_1)={a}, (F_3, A)(e_2)=$\emptyset$, It is clear that (G, A) which is defined as (G, A)(e_1)={b}, (G, A)(e_2)=X, is soft I β-open set and it is not soft -open.

Proof. of the following Theorems are obvious and then omitted.

Theorem 3.1 Let I={$\emptyset_A$} be a soft ideal on a soft topological space (X, τ, A). Then, (i) Soft pre-openness is equivalent to soft I pre-openness, (ii) Soft α-openness is equivalent to soft I α-openness, (iii) Soft β-openness is equivalent to soft I β-openness.

Theorem 3.2 Let I and J be two soft ideals on a soft topological space (X, τ, A). Then, the following statements hold: (i) If J is soft finer than I (i.e, I$\sqsubseteq$J), then every soft I pre-open (resp. I α-open, I

β-open) set, is a soft J pre-open (resp., J α-open, J β-open), (ii) If (F, A) is a soft (I⊓J) pre-open (resp., (I⊓J) α-open, (I⊓J) β-open) set, then it is simultaneously soft I pre-open (resp., I α-open, I β-open) and soft J pre-open (resp., J α-open, J β-open).

Remark 3.2 Clearly, finite soft union of soft I pre-(resp., α-, β-) open sets is so soft I pre-(resp., α-, β-) open, whereas arbitrary soft union of soft pre-(resp., α-, β-) open sets is a soft pre-(resp., α-, β-) open.

Theorem 3.3 Let I be a soft ideal on a soft topological space (X, τ, A). Then, the following statements hold:

(i) Soft union of two soft I pre-open sets is soft I pre-open,

(ii) Soft union of two soft I α-open sets is soft I α-open,

(iii) Soft union of two soft I β-open sets is soft I β-open.

Proof. we prove only (iii) and the rest of the **Proof.** is similar. (iii) Let (F, A) and (G, A) be soft I β-open sets, then there exist soft open sets (U, A), (V, A) such that [(F, A)-Scl(U, A)]∈I, [(U, A)-Scl(F, A)]∈I, [(G, A)-Scl(V, A)]∈I and [(V, A)-Scl(G, A)]∈I. Choose (W, A)=(U, A)⊔(V, A), then (W, A) is soft open set. Hence, [(F, A)-Scl(W, A)]⊑[(F, A)-Scl(U, A)]∈I, and [(G, A)-Scl(W, A)]⊑[(G, A)-Scl(V, A)]∈I. Therefore, [(F, A)⊔(G, A)]-Scl(w, A)∈I. Also, [(U, A)-Scl((F, A)⊔(G, A))]⊑[(U, A)-Scl(F, A)]∈I and [(V, A)-Scl((F, A)⊔(G, A))]⊑[(V, A)-Scl(G, A)]∈I. Hence, [(U, A)⊔(V, A)]- Scl[(F, A)⊔(G, A)]∈I and so [(W, A)-Scl(F, A)⊔(G, A)]∈I. Then, the soft set (F, A)⊔(G, A) is soft I β-open.

Next Example shows that the soft intersection of two soft I pre-open sets is not necessary to be soft I pre-open set.

Example 3.6 Let X={a, b}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and soft ideal I={$\emptyset_A$, (C_1, A), (C_2, A), (C_3, A)} where (F_1, A), (F_2, A), (F_3, A), (C_1, A), (C_2, A) and (C_3, A) are soft sets over X, de?ned as follow (F_1, A)(e_1)={a}, (F_2, A)(e_2)={a}, (F_2, A)(e_1)=X, (F_2, A)(e_2)={b}, (F_3, A)(e_1)={a}, (F_3, A)(e_2)=$\emptyset$, (C_1, A)(e_1)={b}, (C_1, A)(e_2)=$\emptyset$, (C_2, A)(e_1)=$\emptyset$, (C_2, A)(e_2)={b}, (C_3, A)(e_1)={b}, (C_3, A)(e_2)={b}. Let (G, A) and (H, A) be two soft sets such that (G, A)(e_1)={b}, (G, A)(e_2)=X and (H, A)(e_1)={a}, (H, A)(e_2)={a}. It is clear that (G, A) and (H, A) are soft I pre-open sets but their soft intersection (M, A) is not soft I pre-open set.

The soft intersection of two soft I β-open sets need not to be a soft I α-open as shown as the following Example.

Example 3.7 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A)} and I={(C_1, A), (C_2, A), , (C_{15}, A)} where (F_1, A), (F_2, A), (C_1, A), (C_2, A), , (C_{15}, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_2, A)(e_2)={a, c}, (F_2, A)(e_1)={b, c}, (F_2, A)(e_2)={b}, (C_1, A)(e_1)=$\emptyset$, (C_1, A)(e_2)={a, c}, (C_2, A)(e_1)={b}, (C_2, A)(e_2)={a}, (C_3, A)(e_1)={b}, (C_3, A)(e_2)={a, c}, (C_4, A)(e_1)={a}, (C_4, A)(e_2)={a}, (C_5, A)(e_1)={a}, (C_5, A)(e_2)={c}, (C_6, A)(e_1)={a}, (C_6, A)(e_2)=$\emptyset$, (C_7, A)(e_1)=$\emptyset$, (C_7, A)(e_2)={c}, (C_8, A)(e_1)=$\emptyset$, (C_8, A)(e_2)={a}, (C_9, A)(e_1)={b}, (C_9, A)(e_2)=$\emptyset$, (C_{10}, A)(e_1)={b}, (C_{10}, A)(e_2)={c}, (C_{11}, A)(e_1)={a, b}, (C_{11}, A)(e_2)={a}, (C_{12}, A)(e_1)={a}, (C_{12}, A)(e_2)={a, c}, (C_{13}, A)(e_1)={a, b}, (C_{13}, A)(e_2)={a, c}, (C_{14}, A)(e_1)={a, b}, (C_{14}, A)(e_2)={c}, (C_{15}, A)(e_1)={a, b}, (C_{15}, A)(e_2)=$\emptyset$. Let (G_1, A), (G_2, A) be two soft sets such that (G_1, A)(e_1)={a, b}, (G_1, A)(e_2)={b, c}, (G_2, A)(e_1)={a, c}, (G_2, A)(e_2)={a, b}. It is clear that (G_1, A) and (G_2, A) are soft I α-open sets but their soft soft intersection (M, A)(e_1)={a}, (M2, A)(e_2)={b} is not soft I α-open set.

Next Example shows that the soft intersection of two soft I β-open sets need not to be a soft I β-open set as shown in the following Example.

Example 3.8 Let X={a,b,c},A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C, A)} where (F_1, A), (F_2, A), (F_3, A) and (C, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={c}, (F_2, A)(e_1)={c}, (F_2, A)(e_2)=$\emptyset$, (F_3, A)(e_1)={a, c}, (F_3, A)(e_2)={c}, (C, A)(e_1)={c}, (C, A)(e_2)=$\emptyset$, Let (G, A) and (H, A) be two soft I β-open sets such that (G, A)(e_1)={a, b}, (G, A)(e_2)={b, c}, (H, A)(e_1)={b, c}, (H, A)(e_2)={b} but their intersection (M, A)=(G, A)⊓(H,A)is not soft I β-open set.

Theorem 3.4 Let I be a soft ideal on a soft topological space (X, τ, A), then the soft intersection of two soft I pre-(resp., soft I α-, I β-) closed sets is soft I pre-(resp., soft I α-, I β-) closed. Proof Straightforward.

Theorem 3.5 Let I be a soft ideal on a soft topological space (X, τ, A). If (G, A) is a soft open set, then the following assertions hold:

(i) If (F, A) is soft I pre-open set, then the soft intersection (F, A)⊓(G, A) is a soft I pre-open set,

(ii) If (F, A) is soft I α-open set, then the soft intersection (F, A)⊓(G, A) is a soft I α-open set,

(iii) If (F, A) is soft I β-open set, then the soft intersection (F, A)⊓(G, A) is a soft I β-open set.

Proof. we prove only (ii) and the rest of the proof is similar.

(ii) Let (F, A) be a soft I α-open set and (G, A) be a soft open set, then there exist soft open set (H, A) such that [(F, A)-Sint(Scl(H, A))]∈I and [(H, A)-(F, A)]∈I. Hence [((G, A)⊓(F, A))-Sint(Scl(H, A))]∈I and [((G, A)⊓(H, A))-(F, A)]∈I. Since (G, A)⊓Sint(Scl(H, A))=Sint[(G, A)⊓Scl (H, A)]⊑Sint(Scl[(G, A)⊓(H, A)]), then [((G, A)⊓(F, A))-Sint(Scl((G, A)⊓(H, A))]⊑(G, A)⊓(F, A)-(G, A)⊓ Sint(Scl(H, A))=[(G, A)⊓(F, A)-Sint(Scl(H, A))]∈I. Also, [(G, A)⊓(H, A)]-[(G, A)⊓(F, A)]=[(G, A)⊓(H, A)-(F, A)]∈I. Hence there exist soft open set (M, A)=(G, A)⊓(H, A) such that [(G, A)⊓(F, A)-Sint(Scl(M, A))]∈I and [(M, A)-((G, A)⊓(F, A))]∈I. Consequently, (G, A)⊓(F, A) is soft I α-open set.

Theorem 3.6 I be a soft ideal on a soft topological space (X, τ, A). If (G, A) is a soft closed set, then the following assertions hold:

(i) If (F, A) be soft I pre-closed set, then the soft union (F, A)⊔(G, A) is a soft I pre-closed set,

(ii) If (F, A) is soft I α-closed set, then the soft union (F, A)⊔(G, A) is a soft I α- closed set, (iii) If (F, A) is soft I -closed set, then the soft union (F, A)⊔(G, A) is a soft I - closed set.

Proof. Straightforward.

Lemma 3.2 Let I be a soft ideal on a soft topological space (X, τ, A). If Scl(F, A) is soft I pre-open set, then (F, A) is soft I pre-open.

Proof. Let Scl(F, A) is a soft I pre-open set, then there is a soft open set (H, A) such that [Scl(F, A)-(H, A)]∈I, [(H, A)-Scl Scl(F, A)]∈I. Hence, [(F, A) (H, A)]∈I and [(H, A) Scl(F, A)]∈I. This shows that (F, A) is soft I pre-open set.

Lemma 3.3 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is a soft dense set, then it is soft I pre-open.

Proof. Since (F, A) is soft dense, then Scl(F, A)=X_A. Hence X_A-Scl(F, A)=$\emptyset_A$∈I and (F, A)-X_A =$\emptyset_A$∈I. Therefore, (F, A) is soft I pre-open set.

Corollary 3.1 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is soft dense set, then it is soft I β-open.

Proof. Obvious.

Theorem 3.7 Let I be a soft ideal on a soft topological space (X, τ, A). Then, the soft intersection of soft dense set and soft open set is a soft I pre-open.

Proof. Let (F, A) be a soft dense set and (G, A) be a soft open set, then ((F, A)⊓(G, A))-(G, A)=$\emptyset_A$∈I. Since by using Lemma 2.1, Scl(F, A)⊓(G, A)⊑Scl((F, A)⊓(G, A)) and Scl(F, A)=X_A, then (G, A)⊑Scl((F, A)⊓(G, A)) and so (G, A)-Scl((F,A)⊓(G,A))=$\emptyset_A$∈I. Consequently, ((F, A)⊓(G, A)) is a soft I pre-open set.

Corollary 3.2 Let I be a soft ideal on a soft topological space (X, τ, A). Then, the soft intersection of soft dense and soft open set is a soft I β-open set.

Proof. Immediate.

Theorem 3.8 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is a soft I pre-open set and (G, A) is a soft dense set such that (G, A)⊑(F, A), then (G, A) is soft I pre-open set.

Proof. Let (F, A) be a soft I pre-open set, then there exist soft open set (H, A) such that [(F, A)-(H, A)]∈I and [(H, A)-Scl(F, A)]∈I. Since (G, A)⊑(F, A), then [(G, A) (H, A)]∈I. Since (G, A) is soft dense set then Scl(G, A)=X_A, then [(H, A)-Scl(G, A)]∈I for some soft open set (H, A). Hence (G, A) is soft I pre-open set.

Theorem 3.9 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is a soft I pre-open set and (G, A)⊑(F, A)⊑Scl(G, A), then (G, A) is soft I pre β-open set.

Proof. Straightforward.

Theorem 3.10 Let I be a soft ideal on a soft hyper connected topological space (X, τ, A) and (F, A), (G, A) be soft sets. If (F, A)⊑(G, A) and (F, A) is soft I α-open set, then (G, A) is soft I α-open set.

Proof. Let (F, A) be soft I α-open, then there exist soft open set (H, A) such that (F, A)-Sint(Scl(H, A))∈I and (H, A)-(F,A)∈I. Since (F, A)⊑(G, A), then (H, A) (G, A)∈I. Since (X, τ, A) is soft hyper connected space, then Scl(H, A)=X_A. So (G, A) Sint(Scl(H, A))=$\emptyset_A$∈I. Hence, (G, A) is soft I α-open set.

Corollary 3.3 Let I be a soft ideal on a soft hyper connected topological space (X, τ, A). If a soft set (F, A) is soft I α-open, then Scl(F, A) is soft I α-open set.

Proof. In view of Theorem 3.10, the proof is obvious.

Theorem 3.11 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is a soft dense set and (G, A) is a soft open set, then Scl((F, A)⊓(G, A)) is soft I β-open.

Proof. Let (F, A) be a soft dense set, then Scl(F, A)=X_A. Since (G, A) be a soft open set, then Scl(F, A) ⊓(G, A)⊑Scl((F, A)⊓(G, A)). Hence (G, A)⊑Scl((F, A)⊓(G, A)) and (G, A)-Scl((F, A)⊓(G, A))=(G, A)-Scl[Scl((F, A)⊓(G, A))]=$\emptyset_A$∈I. Also, Scl((F, A)⊓(G, A))-Scl (G, A)=$\emptyset_A$∈I Consequently, Scl((F, A)⊓(G, A)) is a soft I β-open set.

Corollary 3.4 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is soft dense set and (G, A) is soft open set, then soft closure of soft I β-open set is soft I β-open.

Proof. Follows immediately by Corollary 3.2, Theorem 3.12.

Theorem 3.12 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is soft I β-open set and (G, A)⊑(F, A)⊑Scl(G, A), then (G, A) is soft I β-open set.

Proof. Straightforward.

Lemma 3.4 Let I be a soft ideal on a soft hyper connected topological space (X, τ, A) and (F, A), (G, A) be soft sets. If (F, A)⊑(G, A) and (F, A) is soft I β-open set, then (G, A) is soft I β-open set.

Proof. Let (F, A) be soft I -open set, then there exist soft open set (H, A) such that (F, A) Scl(H, A)∈I and (H, A) Scl(F, A)∈I. Since (F, A)⊑(G, A), then (H, A) Scl(G, A)∈I. Since (X, τ, A) is soft hyper connected space, then Scl(F, A)=X_A and so (G, A) Scl (H, A)=$\emptyset_A$∈I. So (G, A) is soft I β-open set.

Corollary 3.5 Let I be a soft ideal on a soft hyper connected topological space (X, τ, A). If a soft set (F, A) is soft I -open, then so Scl(F, A).

Proof. Obvious in view of Lemma 3.4.

Corollary 3.6 Let I be a soft ideal on a soft topological space (X, τ, A) and (F, A) be a soft set. If Scl(F, A) is a soft I β-open set, then (F, A) is a soft I β-open. Proof Obvious.

Theorem 3.13 Let I be a soft ideal on a soft topological space (X, τ, A). If (F, A) is a soft I pre-open and soft closed set, then there is soft open set (H, A) such that (F, A)△(H, A)∈I.

Proof. Obvious.

Theorem 3.14 Let I be a soft ideal on a soft locally indiscrete topological space (X, τ, A). Then, a soft set (F, A) is a soft I pre-open, if it is soft I semi-open set.

Proof. Let (F, A) be soft I semi-open set, then there exist a soft open set (H, A) such that [(H, A)-(F, A)]∈I and [(F, A)-Scl(H, A)]∈I. Since (X, τ, A) is soft locally indiscrete space, then Scl(H, A)=(H, A). Hence [(F, A)-(H, A)]∈I and ((H, A)-Scl(F, A))⊑((H, A)-(F, A))∈I for some soft open set (H, A). Thus (F, A) is soft I pre-open set.

Theorem 3.15 Let I be a soft ideal on a soft locally indiscrete topological space (X, τ, A). Then, every soft set is soft I pre-closed.

Proof. Let (F, A) be a soft set. Choose (H, A)=Sint(F, A). Since (X, τ, A) is soft locally indiscrete topological space, then (H, A) is soft closed. Hence, Sint(F, A)-(H, A)∈I and (H, A)-(F, A)∈I i.e, (F, A) is soft I pre-closed set.

Corollary 3.7 Let I be an ideal on a soft locally indiscrete topological space (X, τ, A), then the following statements are hold:

(i)Every singleton set is soft I pre-closed,

(ii)Every soft open set is soft I pre-closed,

(iii)Every soft open set is soft I pre-clopen,

(iv)Every soft I semi-open set is soft I pre-clopen.

Proof. Obvious.

Remark 3.3 Soft I α-open and soft I pre-open sets are independent concepts i.e, there is no any relation between the concepts of soft I α-open set and soft I pre-open set although every soft α-open set is soft pre-open.

The next Example shows that a soft I α-open set is not to be soft I pre-open.

Example 3.9 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A), (C_7, A)} where (F_1, A), (F_2, A), (F_3, A), (C_1, A), (C_2, A), (C_3, A), (C_4, A), (C_5, A), (C_6, A), and (C_7, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={b, c}, (F_2, A)(e_1)={a, b}, (F_2, A)(e_2)={b, c}, (F_3, A)(e_1)=X, (F_3, A)(e_2)={b, c}, (C_1, A)(e_1)={a}, (C_1, A)(e_2)={b, c}, (C_2, A)(e_1)=$\emptyset$, (C_2, A)(e_2)={b, c}, (C_3, A)(e_1)={a}, (C_3, A)(e_2)={b}, (C_4, A)(e_1)={a}, (C_4, A)(e_2)={c}, (C_5, A)(e_1)=$\emptyset$, (C_5, A)(e_2)={b}, (C_6, A)(e_1)=$\emptyset$, (C_6, A)(e_2)={c}, (C_7, A)(e_1)={a}, (C_7, A)(e_2)=$\emptyset$, A soft set (G, A), which is de?ned as (G, A)(e_1)=$\emptyset$, (G, A)(e_2)={a}, is soft I α-open set and it is not soft I pre-open.

The next Example shows that a soft I pre-open set is not to be soft I α-open.

Example 3.10 Let X={a, b, c}, A={α_1, α_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C, A)}, where (F_1, A), (F_2, A), (F_3, A) and (C, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={b, c}, (F_2, A)(e_1)={a, b}, (F_2, A)(e_2)={b, c}, (F_3, A)(e_1)=X, (F_3, A)(e_2)={b, c},(C, A)(e_1)=$\emptyset$, (C, A)(e_2)={a}, A soft set (G, A), which is de?ned as (G, A)(e_1)=$\emptyset$, (G, A)(e_2)=X is soft I pre-open set and it is not soft I α-open.

Lemma 3.5 Let I be a soft ideal on a soft locally indiscrete topological space (X, τ, A). If (F, A) is soft I α-open set, then it is soft I pre-open.

Proof. Straightforward.

Theorem 3.16 Let I be a soft ideal on a soft topological space (X, τ, A). Then, every soft I α-open set is soft I semi-open.

Proof. Let (F, A) be a soft I α-open set then, there exist soft open set (H, A) such that [(F, A)-Sint(Scl(H, A))]∈I and [(H, A)-(F, A)]∈I. Since Sint(Scl(H, A))⊑Scl(H, A), then [(F, A)-Scl(H, A)]∈I.

Hence, there exist soft open set (H, A) such that [(F, A)-Scl(H, A)]∈I and [(H, A)-(F, A)]∈I. Consequently, (F, A) is soft I semi-open set.

Remark 3.4 The next Example shows that a soft I semi-open set need not to be soft I β-open.

Example 3.11 Let X={a, b, c}, A={α_1, α_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C_1, A), (C_2, A), (C_3, A)} where (F_1, A), (F_2, A), (F_3, A), (C_1, A), (C_2, A) and (C_3, A) are soft sets over X, defined as follows: (F_1, A)(e_1)={a}, (F_1, A)(α_2)={c}, (F_2, A)(e_1)={c}, (F_2, A)(e_2)=$\emptyset$, (F_3, A)(e_1)={a, c}, (F_3, A)(e_2)={c}, (C_1, A)(e_1)={a}, (C_1, A)(e_2)={c}, (C_2, A)(e_1)={a}, (C_2, A)(e_2)=$\emptyset$, (C_3, A)(e_1)=$\emptyset$, (C_3, A)(e_2)={c}, It is clear that a soft set (G, A), which is de?ned by (G, A)(e_1)=$\emptyset$, (G, A)(e_2)={a}, is soft I semi-open set and it is not soft I α-open.

Theorem 3.17 Let I be a soft ideal on a soft locally indiscrete topological space (X, τ, A). Then, a soft set (F, A) is soft I α-open, if it is soft I semi-open. Proof Immediate.

Theorem 3.18 Let I be an ideal on extremally disconnected topological space (X, τ, A), then the concepts soft I semi-open and soft I α-open are coincide. Proof Obvious.

Theorem 3.19 Let I be a soft ideal on a soft topological space (X, τ, A), then the following statements hold:

(i) Every soft I semi-open set is soft I β-open,

(ii) Every soft I pre-open set is soft I β-open.

Proof. (i) Let (F, A) be a soft I semi-open set, then there exist soft open set (H, A) such that [(F, A)-Scl(H, A)]∈I and [(H, A)-(F, A)]∈I. Since (F, A)⊑Scl(F, A), then [(H, A)-Scl(F, A)]∈I. Consequently, (F, A) is soft I -open, (ii) Let (F, A) be a soft I pre-open set, then there exist soft open set (H, A) such that [(F, A)-(H, A)]∈I and [(H, A)-Scl(F, A)]∈I. Since (H, A)⊑Scl(H, A), then [(F, A)-Scl(H, A)]∈I. Hence, (F, A) is soft I β-open.

Remark 3.5 The converses of Theorem 3.19 are not true in general as shown in the next Examples 3.12, 3.13.

Example 3.12 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C, A)} where (F_1, A), (F_2, A), (F_3, A) and (C, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={c}, (F_2, A)(e_1)={c}, (F_2, A)(e_2)=$\emptyset$, (F_3, A)(e_1)={a, c}, (F_3, A)(e_2)={c}, (C, A)(e_1)={b}, (C, A)(e_2)=$\emptyset$, A soft set (G, A), which is de?ned as (G, A)(e_1)={a}, (G, A)(e_2)={b}, is soft I β-open set and it is not soft I semi-open.

Example 3.13 Let X={a, b, c}, A={e_1, e_2}, τ={$\emptyset_A$, X_A, (F_1, A), (F_2, A), (F_3, A)} and I={$\emptyset_A$, (C, A)} where (F_1, A), (F_2, A), (F_3, A) and (C, A) are soft sets over X, defined as follow: (F_1, A)(e_1)={a}, (F_1, A)(e_2)={c}, (F_2, A)(e_1)={c}, (F_2, A)(e_2)=$\emptyset$, (F_3, A)(e_1)={a, c}, (F_3, A)(e_2)={c}, (C, A)(e_1)=$\emptyset$, (C, A)(e_2)={a}, A soft set (G, A), which is defined as (G, A)(e_1)={b}, (G, A)(e_2)={c}, is soft I β-open set and it is not soft I pre-open.

Remark 3.6 In view of Theorems 3.16, 3.19 and Lemmas 2.2, 3.1, the next implications are true.

The proof of the next theorems is obvious and then omitted.

Theorem 3.20 Let I be a soft ideal on a soft indiscrete topological space(X, τ, A), then each soft I β-open set is soft I pre-open.

Theorem 3.21 Let I be a soft ideal on a soft locally indiscrete topological space (X, τ, A), then every soft I β-open set is I pre-open.

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