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Some types of pairwise soft open (continuous) mappings and some related results

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Abstract In this paper, we introduce and study some new notions in soft bitopological spaces such as pairwise soft continuous mappings, $p\Lambda$ (resp. $p\lambda$, gp, pl)-soft continuous mappings, pairwise open (closed) soft mappings and pairwise soft homeomorphism mappings. Moreover, characterizations of these notions are obtained. In addition that, we investigate the behavior of pairwise soft separation axioms under these types of mappings. Furthermore, the relationships among these notions are studied.

Key Words Soft set; Soft topology; Soft bitopological spaces; pairwise soft continuous; $p\lambda$-soft continuous
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1 Introduction

Molodtsov [15] initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory to several directions such as smoothness of functions, game theory, Riemann Integration, and theory of measurement. In recent years, development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parameterized expressed by a soft set. Shabir and Naz [21] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. I. Zorlutuna et al. [24] studied some new properties of soft continuous mappings and gave some new characterizations of sot continuous, soft open, soft closed mappings and also soft homeomorphisms. Ittanagi [7] introduced the concept of soft bitopological space and studied some types of soft separation axiom for soft bitopological spaces from his point of view. Kandil et al. [[10],[12]]introduced some structures of soft bitopological spaces. Kandil et al. [8] introduced the concept of generalized pairwise closed soft sets and the associated pairwise soft separation axiom namely, PSR_0^*

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and $PST^*_{\frac{1}{2}}$. Recently, Kandil et al. [11] introduced some types of pairwise soft separation axioms, namely, PST^*_0, PST^*_1, PST^*_2 and PSR^*_1. They studied the characterization and implications among these types of separation axioms.

In the present paper, we will attempt to construct the basic theories about some types of Pairwise soft continuous mappings namely, pairwise soft continuous mappings, $p\Lambda$ (resp. $p\lambda$, gp, pl)-soft continuous mappings and open (closed)soft mappings between two soft bitopological spaces. The rest of this paper is organized as follows. In section (2) we introduced briefly the notions of soft set,soft topology, soft bitopological spaces, soft mapping and some related topics. In Section (3), based on some types of soft continuous mappings, we define pairwise soft continuous mapping from one soft bitopological space to another soft bittopological space and give some characterizations of these types of soft continuous mappings. We also investigate the behavior of pairwise soft separation axioms under such types of soft continuous mappings. Section (4) gives the concept of p-soft open(closed) mappings. Some related properties are studied. The last section summarizes the conclusions.

2 Preliminaries

In this section, we briefly review some concepts and some related results of soft set, soft topological space and soft bitopological space which are needed to used in current paper. For more details about these concepts you can see in [[7], [2], [3], [4], [5], [6], [8], [10], [11], [15], [16], [17], [20], [21], [22], [23]].

Let X be an initial universe, E be a set of parameters and $P(X)$ be the power set of X.

Definition 2.1. *[17] A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. A soft set can also be defined by the set of ordered pairs*

$$(F, E) = \{(e, F(e)) : e \in E, \quad F : E \to P(X)\}.$$

From now on, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.
For two soft sets (F, E), $(G, E) \in SS(X)_E$, (F, E) is called a soft subset of (G, E), denoted by $(F, E)\tilde{\subseteq}(G, E)$, if $F(e) \subseteq G(e), \forall e \in E$. In this case, (G, E) is called a soft superset of (F, E). In addition, the union of soft sets (F, E) and (G, E), denoted by $(F, E)\tilde{\cup}(G, E)$, is the soft set (H, E) which defined as $H(e) = F(e) \cup G(e)$, $\forall e \in E$. Moreover, the intersection of soft sets (F, E) and (G, E), denoted by $(F, E)\tilde{\cap}(G, E)$, is the soft set (M, E) which defined as $M(e) = F(e) \cap G(e)$, $\forall e \in E$. The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined as $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, $\forall\, e \in E$. The difference of soft sets (F, E) and (G, E), denoted by $(F, E) \setminus (G, E)$, is the soft set (H, E), which defined as $H(e) = F(e) \setminus G(e)$, $\forall\, e \in E$. Clearly, $(F, E)\setminus(G, E) = (F, E)\tilde{\cap}(G, E)^c$. A soft set (F, E) is called a null soft set, denoted by $(\tilde{\phi}, E)$, if $F(e) = \phi$, $\forall\, e \in E$. Moreover, a soft set (F, E) is called an absolute soft set, denoted by $(\tilde{X}, E)$, if $F(e) = X$, $\forall e \in E$. Clearly, we have $(\tilde{\phi}, E)^c = (\tilde{X}, E)$ and $(\tilde{X}, E)^c = (\tilde{\phi}, E)$. Moreover, a soft set (G, E) is said to be a finite soft set if $G(e)$ is a finite set for all $e \in E$. Otherwise, it is called an infinite soft set.

Definition 2.2 ([1],[16],[18],[22]). *A soft set $(F, E) \in SS(X)_E$ is said to be a soft point in $(\tilde{X}, E)$ if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E \setminus \{e\}$. This soft point is denoted by (x_e, E) or x_e, i.e.,*
$x_e : E \to P(X)$ is a mapping defined as

$$x_e(a) = \begin{cases} \{x\} & if \;\; e = a, \\ \phi & if \;\; e \neq a \end{cases} \qquad \text{for all } a \in E.$$

A soft point (x_e, E) is said to be belonging to the soft set (G, E), denoted by $x_e \tilde{\in} (G, E)$, if $x_e(e) \subseteq G(e)$, i.e., $\{x\} \subseteq G(e)$. Clearly, $x_e \tilde{\in} (G, E)$ if and only if $(x_e, E) \tilde{\subseteq} (G, E)$. In addition, two soft points x_{e_1}, y_{e_2} over X are said to be equal if $x = y$ and $e_1 = e_2$. Thus, $x_{e_1} \neq y_{e_2}$ iff $x \neq y$ or $e_1 \neq e_2$.

The family of all soft points in $(\tilde{X}, E)$ is denoted by $\xi(X)_E$.

Proposition 2.3. *[22] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as a union of all soft points belonging to it, i.e., $(G, E) = \tilde{\bigcup}\{(x_e, E) : x_e \tilde{\in} (G, E)\}$.*

Proposition 2.4. *[22] Let (G, E), (H, E) be two soft sets over X. Then,*

(1) $x_e \tilde{\in} (G, E) \Leftrightarrow x_e \tilde{\notin} (G, E)^c$.

(2) $x_e \tilde{\in} (G, E) \tilde{\cup} (H, E) \Leftrightarrow x_e \tilde{\in} (G, E)$ *or* $x_e \tilde{\in} (H, E)$.

(3) $x_e \tilde{\in} (G, E) \tilde{\cap} (H, E) \Leftrightarrow x_e \tilde{\in} (G, E)$ *and* $x_e \tilde{\in} (H, E)$.

(4) $(G, E) \tilde{\subseteq} (H, E) \Leftrightarrow \; [x_e \tilde{\in} (G, E) \Rightarrow x_e \tilde{\in} (H, E)]$.

For more details for soft point you can see in [[16],[22],[18]].

Definition 2.5. *[21], [19] Let η be a collection of soft sets over a universe X with a fixed set of parameters E, i.e., $\eta \subseteq SS(X)_E$. The collection η is called a soft topology on X if it satisfies the following axioms:*

(1) $(\tilde{X}, E), (\tilde{\phi}, E) \in \eta$,

(2) *The union of any number of soft sets in η belongs to η,*

(3) *The intersection of any two soft sets in η belongs to η.*

The triple (X, η, E) is called a soft topological space. Any member of η is said to be an open soft set in (X, η, E). A soft set (F, E) over X is said to be a closed soft set in (X, η, E), if its complement $(F, E)^c$ is an open soft set in (X, η, E).

We denote the family of all closed soft sets by η^c.

Definition 2.6. *[13],[23] Let $SS(X)_E$ and $SS(Y)_K$ be two families of soft sets. Let $u : X \to Y$ and $p : E \to K$ be mappings. We define a soft mapping $f_{pu} : SS(X)_E \to SS(Y)_K$ as:*

(i) *If $(G, E) \in SS(X)_E$. Then, the image of (G, E) under f_{pu}, written as $f_{pu}(G, E) = (f_{pu}(G), p(E))$, is a soft set in $SS(Y)_K$ such that*

$$f_{pu}(G,E)(k) = \begin{cases} \bigcup_{e\in p^{-1}(k)} u[G(e)] & if \;\; p^{-1}(k) \neq \phi, \\ \phi & if \;\; p^{-1}(k) = \phi \end{cases}$$

for all $k \in K$.

(ii) *If* $(H,K) \in SS(Y)_K$*. Then, the inverse image of* (H,K) *under* f_{pu}*, written as* $f_{pu}^{-1}(H,K) = (f_{pu}^{-1}(H),E)$*, is a soft set in* $SS(X)_E$ *such that*

$$f_{pu}^{-1}(H,K)(e) = u^{-1}[H(p(e))] \;\; \forall \; e \in E.$$

Theorem 2.7. *[13] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu} : SS(X)_E \to SS(Y)_K$ *we have the following:*

(1) $f_{pu}(\tilde{\phi},E) = (\tilde{\phi},K)$ *and* $f_{pu}(\tilde{X},E)\tilde{\subseteq}(\tilde{Y},K)$.

(2) $f_{pu}[\tilde{\bigcup}_{i\in I}(G_i,E)] = \tilde{\bigcup}_{i\in I} f_{pu}(G_i,E)$ *and* $f_{pu}[\tilde{\bigcap}_{i\in I}(G_i,E)]\tilde{\subseteq}\tilde{\bigcap}_{i\in I} f_{pu}(G_i,E)$.

(3) $(G,E)\tilde{\subseteq}(M,E) \Rightarrow f_{pu}(G,E)\tilde{\subseteq}f_{pu}(M,E)$.

Theorem 2.8. *[13],[14] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu} : SS(X)_E \to SS(Y)_K$ *we have the following:*

(1) $f_{pu}^{-1}(\tilde{\phi},K) = (\tilde{\phi},E)$ *and* $f_{pu}^{-1}(\tilde{Y},K) = (\tilde{X},E)$.

(2) $f_{pu}^{-1}[\tilde{\bigcup}_{i\in I}(H_i,K)] = \tilde{\bigcup}_{i\in I} f_{pu}^{-1}(H_i,K)$ *and* $f_{pu}^{-1}[\tilde{\bigcap}_{i\in I}(H_i,K)] = \tilde{\bigcap}_{i\in I} f_{pu}^{-1}(H_i,K)$.

(3) $(H,K)\tilde{\subseteq}(N,K) \Rightarrow f_{pu}^{-1}(H,K)\tilde{\subseteq}f_{pu}^{-1}(N,K)$.

Definition 2.9. *[23] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. A soft mapping* $f_{pu} : SS(X)_E \to SS(Y)_K$ *is called soft surjective(soft injective) mapping if* u,p *are surjective (injective) mappings, respectively. A soft mapping which is a soft surjective and soft injective mapping is called a soft bijection mapping.*

Theorem 2.10. *[23] Let* $SS(X)_E$ *and* $SS(Y)_K$ *be two families of soft sets. For a soft mapping* $f_{pu} : SS(X)_E \to SS(Y)_K$ *we have the following:*

(1) $f_{pu}^{-1}[(\tilde{Y},K) \setminus (H,K)] = (\tilde{X},E) \setminus [f_{pu}^{-1}(H,K)]$ *for any* $(H,K) \in SS(Y)_K$.

(2) $(G,E)\tilde{\subseteq}f_{pu}^{-1}[f_{pu}(G,E)]$ *for any* $(G,E) \in SS(X)_E$*. If* f_{pu} *is a soft injective, the equality holds.*

(3) $f_{pu}[f_{pu}^{-1}(H,K)]\tilde{\subseteq}(H,K)$ *for any* $(H,K) \in SS(Y)_K$*. If* f_{pu} *is a soft surjective, the equality holds.*

Definition 2.11. *[24] Let* (X,η,E) *and* (Y,σ,K) *be two soft topological spaces. A soft mapping* $f_{pu} : (X,\eta,E) \to (Y,\sigma,K)$ *is said to be a soft continuous if the inverse image of any open soft set in* (Y,σ,K) *is an open soft set in* (X,η,E)*, i.e.,* $f_{pu}^{-1}(H,K) \in \eta$ *for any* $(H,K) \in \sigma$.

Definition 2.12. *[7] A quadrable system* (X,η_1,η_2,E) *is called a soft bitopological space [briefly, sbts], where* η_1*,* η_2 *are arbitrary soft topologies on* X *with a fixed set of parameters* E.

Definition 2.13. *[10] Let (X, η_1, η_2, E) be a sbts. A soft set (G, E) over X is said to be a pairwise open soft set in (X, η_1, η_2, E) [briefly, p-open soft set] if there exist an open soft set (G_1, E) in η_1 and an open soft set (G_2, E) in η_2 such that $(G, E) = (G_1, E)\tilde{\cup}(G_2, E)$. A soft set (G, E) over X is said to be a pairwise closed soft set in (X, η_1, η_2, E) [briefly, p-closed soft set] if its complement is a p-open soft set in (X, η_1, η_2, E). Clearly, a soft set (F, E) over X is a p-closed soft set in (X, η_1, η_2, E) if there exist an η_1-closed soft set (F_1, E) and an η_2-closed soft set (F_2, E) such that $(F, E) = (F_1, E)\tilde{\cap}(F_2, E)$.*

The family of all p-open (p-closed) soft sets in a sbts (X, η_1, η_2, E) is denoted by η_{12} (η_{12}^c), respectively.

Theorem 2.14. *[10] Let (X, η_1, η_2, E) be a sbts. The family of all p-open soft sets, denoted by η_{12} or $\eta_1 \tilde{\bigsqcup} \eta_2$ is a supra soft topology on X, where*

$$\eta_{12} = \{(G, E) = (G_1, E)\tilde{\cup}(G_2, E) : (G_i, E) \in \eta_i, i = 1, 2\}.$$

The triple (X, η_{12}, E) is the supra soft topological space associated to the sbts (X, η_1, η_2, E).

Lemma 2.15. *[8] For any sbts (X, η_1, η_2, E), we have $(X, \eta_1^e, \eta_2^e, E)$ is an ordinary bitopological space, for all $e \in E$,where $\eta_i^e = \{G(e) : (G, E) \in \eta_i\}$, $i = 1, 2$, and $\eta_1^e \tilde{\bigsqcup} \eta_2^e = \eta_{12}^e = \{M(e) : (M, E) \in \eta_{12}\}$.*

Definition 2.16. *[10] Let (X, η_1, η_2, E) be a sbts and let $(G, E) \in SS(X)_E$. The pairwise soft closure of (G, E), denoted by $scl_{\eta_{12}}(G, E)$, is defined by*

$scl_{\eta_{12}}(G, E) = \tilde{\bigcap}\{(F, E) \in \eta_{12}^c : (G, E)\tilde{\subseteq}(F, E)\}$.

Clearly, $scl_{\eta_{12}}(G, E)$ is the smallest p-closed soft set contains (G, E).For more details about the properties of pairwise soft closure operator see in [10].

Definition 2.17. *[10] Let (X, η_1, η_2, E) be a sbts and let $(G, E) \in SS(X)_E$. The pairwise soft interior of (G, E), denoted by $sint_{\eta_{12}}(G, E)$, is defined by*

$sint_{\eta_{12}}(G, E) = \tilde{\bigcup}\{(H, E) \in \eta_{12} : (H, E)\tilde{\subseteq}(G, E)\}$.

Clearly, $sint_{\eta_{12}}(G, E)$ is the largest p-open soft set contained in (G, E). For more details about the properties of pairwise soft interior operator you can see see [10].

Theorem 2.18. *[10] Let (X, η_1, η_2, E) be a sbts and $(G, E) \in SS(X)_E$. Then,*

(1) $sint_{\eta_{12}}(G, E) = [scl_{\eta_{12}}(G, E)^c]^c$.

(2) $scl_{\eta_{12}}(G, E) = [sint_{\eta_{12}}(G, E)^c]^c$.

Definition 2.19. *[8] Let (X, η_1, η_2, E) be a sbts. A soft set (G, E) is said to be a generalized pairwise closed soft set [briefly, gp-closed soft set] if $scl_{\eta_{12}}(G, E)\tilde{\subseteq}(H, E)$ whenever $(G, E)\tilde{\subseteq}(H, E)$ and (H, E) is a p-open soft set. A soft set (M, E) is called a generalized pairwise open soft set [briefly, gp-open soft set] if its complement is a gp-closed soft set.*

The family of all gp-closed (gp-open) soft sets on X denoted by $GpC(X, \eta_1, \eta_2)_E$ or $GpC(X)_E$ [$GpO(X, \eta_1, \eta_2)_E$ or $GpO(X)_E$], respectively.

Theorem 2.20. *[8] Let (X, η_1, η_2, E) be a sbts and $(G, E) \in SS(X)_E$. Then,*

every p-closed soft set is a gp-closed soft set.

Definition 2.21. *[9] A soft set (G, E) is said to be a pairwise locally closed soft set in a sbts (X, η_1, η_2, E) [briefly, pl-closed soft set] if $(G, E) = (F, E)\tilde{\cap}(H, E)$, where (F, E) is a p-closed soft set and (H, E) is a p-open soft set. The complement of pl-closed soft set is called pl-open soft set. The family of all pl-closed soft sets (pl-open soft sets) we denoted by $PLCS(X, \eta_1, \eta_2)_E(PLOS(X, \eta_1, \eta_2)_E)$, respectively.*

Theorem 2.22. *[9] Let (X, η_1, η_2, E) be a sbts. Then,*
every p-open (p-closed) soft set is p-locally closed soft set.

Definition 2.23. *[10] Let (X, η_1, η_2, E) be a sbts and let $(G, E) \in SS(X)_E$. The pairwise soft kernel of (G, E) [briefly, $sker_{\eta_{12}}(G, E)$], is the intersection of all p-open soft supersets of (G, E), i.e.,*

$$sker_{\eta_{12}}(G, E) = \tilde{\bigcap}\{(H, E) \in \eta_{12} : (G, E)\tilde{\subseteq}(H, E)\}.$$

Definition 2.24. *[10] A soft set (G, E) is said to be a pairwise Λ- soft set in a sbts (X, η_1, η_2, E) [briefly, pΛ-soft set] if $sker_{\eta_{12}}(G, E) = (G, E)$.*

Theorem 2.25. *[10] Every p-open soft set is a pΛ-soft set.*

Theorem 2.26. *[10] Let (X, η_1, η_2, E) be a sbts. Then, the class of all pΛ-soft sets is an Alexandroff soft topology on X. This soft topology denoted by $\eta_{p\Lambda}$. The triple $(X, \eta_{p\Lambda}, E)$ is the soft topological space associated to the sbts (X, η_1, η_2, E), induced by the family of all pΛ-soft sets.*

Theorem 2.27. *[10] Let (X, η_1, η_2, E) be a sbts. Then,*

$$\eta_1 \cup \eta_2 \subseteq \eta_{12} \subseteq \eta_{p\Lambda} \subseteq SS(X)_E.$$

Definition 2.28. *[10] A soft set (G, E) is said to be a pairwise λ-closed soft set in a sbts (X, η_1, η_2, E) [briefly, pλ-closed soft set] if $(G, E) = (F, E)\tilde{\cap}(H, E)$, where (F, E) is a p-closed soft set and (H, E) is a pΛ-soft set. The family of all pλ-closed soft sets we denoted by $P\lambda CS(X, \eta_1, \eta_2)_E$.*

Theorem 2.29. *[10] Let (X, η_1, η_2, E) be a sbts. Then,*

(1) *Every p-closed soft set is a pλ-closed soft set.*

(2) *Every pΛ-soft set is a pλ-closed soft set.*

Theorem 2.30. *[8] Let (X, η_1, η_2, E) be a sbts and let $(F, E) \in SS(X)_E$. Then,*
the soft set (F, E) is a p-closed soft set in a sbts (X, η_1, η_2, E) iff it is both gp-closed soft set and pλ-closed soft set in (X, η_1, η_2, E).

Definition 2.31. *[8] A sbts (X, η_1, η_2, E) is said to be a pairwise soft R_0^* [briefly, PSR_0^*] if*

$$x_\alpha \tilde{\in} scl_{\eta_{12}}(y_\beta, E) \Rightarrow y_\beta \tilde{\in} scl_{\eta_{12}}(x_\alpha, E),$$

where $x_\alpha, y_\beta \in \xi(X)_E$.

Definition 2.32. *[11] A sbts (X, η_1, η_2, E) is said to be a pairwise soft T_0^*[briefly, PST_0^*] if for each x_α, $y_\beta \in \xi(X)_E$ with $x_\alpha \neq y_\beta$, there exists $(G, E) \in \eta_{12}$ such that $x_\alpha \tilde{\in}(G, E)$, $y_\beta \tilde{\notin}(G, E)$ or $y_\beta \tilde{\in}(G, E)$, $x_\alpha \tilde{\notin}(G, E)$.*

Lemma 2.33. *[11] Let (X, η_1, η_2, E) be a sbts. Then,*

(X, η_1, η_2, E) is a PST_0^* if and only if for all $x_\alpha, y_\beta \in \xi(X)_E$, $x_\alpha \neq y_\beta$ there exists $(G, E) \in \eta_{12} \cup \eta_{12}^c$ such that $x_\alpha \tilde{\in} (G, E)$ and $y_\beta \tilde{\notin} (G, E)$.

Definition 2.34. *[8] A sbts (X, η_1, η_2, E) is called a pairwise soft $T_{\frac{1}{2}}^*$ [briefly, $PST_{\frac{1}{2}}^*$] if every gp-closed soft set is a p-closed soft set.*

Theorem 2.35. *[8] Let (X, η_1, η_2, E) be a sbts. Then, (X, η_1, η_2, E) is a $PST_{\frac{1}{2}}^*$ if and only if every soft point either p-open soft set or p-closed soft set.*

Definition 2.36. *[11] A sbts (X, η_1, η_2, E) is said to be a pairwise soft T_1^* [briefly PST_1^*] if for each x_α, $y_\beta \in \xi(X)_E$ with $x_\alpha \neq y_\beta$, there exist (G, E), $(H, E) \in \eta_{12}$ such that $x_\alpha \tilde{\in} (G, E)$, $y_\beta \tilde{\notin} (G, E)$ and $y_\beta \tilde{\in} (H, E)$, $x_\alpha \tilde{\notin} (H, E)$.*

Definition 2.37. *[11] A sbts (X, η_1, η_2, E) is said to be a pairwise soft T_2^* [briefly, PST_2^*] if $\forall\, x_\alpha$, $y_\beta \in \xi(X)_E$, $x_\alpha \neq y_\beta$, there exist (G, E), $(H, E) \in \eta_{12}$ such that $x_\alpha \tilde{\in} (G, E)$, $y_\beta \tilde{\in} (H, E)$ and $(G, E) \tilde{\cap} (H, E) = (\tilde{\phi}, E)$.*

3 Decompositions of pairwise soft continuous

Definition 3.1. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. A soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is said to be a pairwise soft continuous [briefly, p-soft continuous] if the inverse image of any p-open soft set in $(Y, \sigma_1, \sigma_2, K)$ is a p-open soft set in (X, η_1, η_2, E), i.e., $f_{pu}^{-1}(H, K) \in \eta_{12}$ for any $(H, K) \in \sigma_{12}$.*

Theorem 3.2. *Let $f_{pu} : (X, \eta_i, E) \to (Y, \sigma_i, K)$, $i = 1, 2$, be a soft continuous mappings. Then, the soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous.*

Proof. Let $(H, K) \in \sigma_{12}$. Then there exist $(H_1, K) \in \sigma_1$ and $(H_2, K) \in \sigma_2$ such that $(H, K) = (H_1, K) \tilde{\cup} (H_2, K) \Rightarrow f_{pu}^{-1}(H, K) = f_{pu}^{-1}[(H_1, K) \tilde{\cup} (H_2, K)]$

$\Rightarrow f_{pu}^{-1}(H, K) = f_{pu}^{-1}(H_1, K) \tilde{\cup} f_{pu}^{-1}(H_2, K)$ [by Theorem 2.8 (2)]. Since $f_{pu} : (X, \eta_i, E) \to (Y, \sigma_i, K)$, $i = 1, 2$, are soft continuous mappings, then $f_{pu}^{-1}(H_1, K) \in \eta_1$ and $f_{pu}^{-1}(H_2, K) \in \eta_2$. It follows that $f_{pu}^{-1}(H, K) = f_{pu}^{-1}(H_1, K) \tilde{\cup} f_{pu}^{-1}(H_2, K) \in \eta_{12}$. Therefore, $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous mapping.

Remark 3.3. *If $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous, then the soft mappings $f_{pu} : (X, \eta_i, E) \to (Y, \sigma_i, K)$, $i = 1, 2$, need not be a soft continuous as shown by the following example.*

Example 3.4. *Let $X = \{x, y, z\}$, $Y = \{a, b, c\}$, $E = \{e_1, e_2\}$ and $K = \{k_1, k_2\}$.*

Let

$$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E)\},$$
$$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H, E)\},$$

where

$$(G, E) = \{(e_1, \{x, y\}), (e_2, \{x\})\},$$

$(H,E) = \{(e_1, \{y,z\}), (e_2, \{x,y\})\}$,

and let

$\sigma_1 = \{(\tilde{\phi}, K), (\tilde{Y}, K), (M,K)\}$,

$\sigma_2 = \{(\tilde{\phi}, K), (\tilde{Y}, K), (F,K)\}$,

where

$(M,K) = \{(k_1, \{a,c\}), (k_2, \{a,b\})\}$,

$(F,K) = \{(k_1, \{a,b\}), (k_2, \{b\})\}$.

Then (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ are soft bitopological spaces.

Moreover,

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G,E), (H,E), (P,E)\}$

where

$(P,E) = (G,E)\tilde{\cup}(H,E) = \{(e_1, X), (e_2, \{x,y\})\}$ *and*

$\sigma_{12} = \{(\tilde{\phi}, K), (\tilde{Y}, K), (F,K), (M,K), (N,E)\}$

where

$(N,E) = (F,E)\tilde{\cup}(M,E) = \{(k_1, Y), (k_2, \{a,b\})\}$.

Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping defined by

$u(x) = b, u(y) = \{a\}, u(z) = c$, *and*

$p(e_1) = k_1, p(e_2) = k_2$.

It is easy to prove that

$f_{pu}(x_{e_1}, E) = (b_{k_1}, K)$, $f_{pu}(x_{e_2}, E) = (b_{k_2}, K)$,

$f_{pu}(y_{e_1}, E) = (a_{k_1}, K)$, $f_{pu}(y_{e_2}, E) = (a_{k_2}, K)$,

$f_{pu}(z_{e_1}, E) = (c_{k_1}, K)$, $f_{pu}(z_{e_2}, E) = (c_{k_2}, K)$.

Furthermore, we can prove that

$f_{pu}^{-1}(M,K) = (H,E)$, $f_{pu}^{-1}(F,K) = (G,E)$, $f_{pu}^{-1}(N,K) = (P,E)$, $f_{pu}^{-1}(\tilde{Y}, K) = (\tilde{X}, E)$, *and* $f_{pu}^{-1}(\tilde{\phi}, K) = (\tilde{\phi}, E)$. *Hence, f_{pu} is a p-soft continuous but it is clear that $f_{pu} : (X, \eta_1, E) \to (Y, \sigma_1, K)$ and $f_{pu} : (X, \eta_2, E) \to (Y, \sigma_2, K)$ are not soft continuous mappings.*

Theorem 3.5. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two sbts and let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a p-soft continuous mapping. Then, $u : (X, \eta_1^e, \eta_2^e) \to (Y, \sigma_1^k, \sigma_2^k)$ is a p-continuous mapping $\forall$ $(e, p(e) = k) \in E \times K$.*

Proof. Let $A \in \sigma_{12}^k$. Then, by Lemma 2.15, there exists $(H,K) \in \sigma_{12}$ such that $(H,K)(k) = A$. Since f_{pu} is a p-soft continuous, then $f_{pu}^{-1}(H,K) \in \eta_{12}$, implies $f_{pu}^{-1}(H,K)(e) \in \eta_{12}^e$. But by Definition 2.6 we have

$$\begin{aligned} f_{pu}^{-1}(H,K)(e) &= u^{-1}[(H,K)(p(e))] \\ &= u^{-1}[(H,K)(k)] \\ &= u^{-1}(A). \text{ It follows that } u^{-1}(A) \in \eta_{12}^e. \text{ Hence, } u \text{ is } p\text{-continuous.} \end{aligned}$$

Theorem 3.6. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping. Then, the following statements are equivalent:*

(1) f_{pu} *is a p-soft continuous.*

(2) $f_{pu}^{-1}(F,K) \in \eta_{12}^c$ *for all* $(F,K) \in \sigma_{12}^c$.

(3) $f_{pu}[scl_{\eta_{12}}(G,E)] \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(G,E)]$ *for any* $(G,E) \in SS(X)_E$.

(4) $scl_{\eta_{12}}[f_{pu}^{-1}(H,K)] \tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}(H,K)]$ *for any* $(H,K) \in SS(Y)_K$.

(5) $f_{pu}^{-1}[sint_{\sigma_{12}}(H,K)] \tilde{\subseteq} sint_{\eta_{12}}[f_{pu}^{-1}(H,K)]$ *for any* $(H,K) \in SS(Y)_K$.

Proof. (1) $\Rightarrow$ (2): Let $(F,K) \in \sigma_{12}^c$. Then, $(F,K)^c \in \sigma_{12}$ it follows by (1) that $f_{pu}^{-1}(F,K)^c$ is a p-open soft set in (X,η_1,η_2,E). But, by Theorem 2.10 (1), $f_{pu}^{-1}(F,K)^c = [f_{pu}^{-1}(F,K)]^c$. Therefore, $f_{pu}^{-1}(F,K)$ is a p-closed soft set in (X,η_1,η_2,E), i.e., $f_{pu}^{-1}(F,K) \in \eta_{12}^c$.

(2) $\Rightarrow$ (3): Let $(G,E) \in SS(X)_E$. Then,

$$\begin{aligned} f_{pu}(G,E) \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(G,E)] &\Rightarrow f_{pu}^{-1} f_{pu}(G,E) \tilde{\subseteq} f_{pu}^{-1} scl_{\sigma_{12}}[f_{pu}(G,E)] \text{ [by Theorem 2.8 (3)]} \\ &\Rightarrow (G,E) \tilde{\subseteq} f_{pu}^{-1} scl_{\sigma_{12}}[f_{pu}(G,E)] \quad \text{[by Theorem 2.10 (2)]} \\ &\Rightarrow scl_{\eta_{12}}(G,E) \tilde{\subseteq} f_{pu}^{-1} scl_{\sigma_{12}}[f_{pu}(G,E)] \quad \text{[from (2)]} \\ &\Rightarrow f_{pu}[scl_{\eta_{12}}(G,E)] \tilde{\subseteq} f_{pu} f_{pu}^{-1} scl_{\sigma_{12}}[f_{pu}(G,E)] \\ &\Rightarrow f_{pu}[scl_{\eta_{12}}(G,E)] \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(G,E)] \quad \text{[by Theorem 2.10 (3)].} \end{aligned}$$

(3) $\Rightarrow$ (4): Let $(H,K) \in SS(Y)_K$. Then, $f_{pu}^{-1}(H,K) \in SS(X)_E$ it follows from (3) that

$$f_{pu}[scl_{\eta_{12}}[f_{pu}^{-1}(H,K)]] \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}[f_{pu}^{-1}(H,K)]].$$

Since $f_{pu}[f_{pu}^{-1}(H,K)] \tilde{\subseteq} (H,K)$, then $scl_{\sigma_{12}}[f_{pu}[f_{pu}^{-1}(H,K)]] \tilde{\subseteq} scl_{\sigma_{12}}(H,K)$. Therefore,

$$f_{pu}[scl_{\eta_{12}}[f_{pu}^{-1}(H,K)]] \tilde{\subseteq} scl_{\sigma_{12}}(H,K)$$

it follows that $f_{pu}^{-1} f_{pu}[scl_{\eta_{12}}[f_{pu}^{-1}(H,K)]] \tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}(H,K)]$.
Hence, $scl_{\eta_{12}}[f_{pu}^{-1}(H,K)] \tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}(H,K)]$.

(4) $\Rightarrow$ (5): It is worth noting that for any $(G,E) \in SS(X)_E$, $(G,E)^c = (\tilde{X},E) \setminus (G,E)$, and for any $(H,K) \in SS(Y)_K$, $(H,K)^c = (\tilde{Y},K) \setminus (H,K)$. Now, let $(H,K) \in SS(Y)_K$. Then $(\tilde{Y},K) \setminus (H,K) \in SS(Y)_K$. It follows from (4) that
$scl_{\eta_{12}}[f_{pu}^{-1}[(\tilde{Y},K) \setminus (H,K)]] \tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y},K) \setminus (H,K)]]$ which gives

$(\tilde{X},E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y},K) \setminus (H,K)]] \tilde{\subseteq} (\tilde{X},E) \setminus scl_{\eta_{12}}[f_{pu}^{-1}[(\tilde{Y},K) \setminus (H,K)]]$................$(*)$.

$$\begin{aligned} \text{Now, } f_{pu}^{-1}[sint_{\sigma_{12}}(H,K)] &= f_{pu}^{-1}[(\tilde{Y},K) \setminus scl_{\sigma_{12}}[(\tilde{Y},K) \setminus (H,K)]] \text{[by Theorem 2.18]} \\ &= (\tilde{X},E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y},K) \setminus (H,K)]] \text{ [by Theorem 2.10 (1)]} \\ &\tilde{\subseteq} (\tilde{X},E) \setminus scl_{\eta_{12}}[f_{pu}^{-1}[(\tilde{Y},K) \setminus (H,K)]] \text{ [from } (*)\text{]} \\ &= (\tilde{X},E) \setminus scl_{\eta_{12}}[(\tilde{X},E) \setminus f_{pu}^{-1}(H,K)] \quad \text{[by Theorem 2.10 (1)]} \\ &= sint_{\eta_{12}} f_{pu}^{-1}(H,K). \end{aligned}$$

Consequently, $f_{pu}^{-1}[sint_{\sigma_{12}}(H,K)]\tilde{\subseteq}sint_{\eta_{12}}[f_{pu}^{-1}(H,K)]$.

(5) $\Rightarrow$ (1): Let $(M,K)\in\sigma_{12}$. Then $sint_{\sigma_{12}}(M,K)=(M,K)$ it follows from (5) that

$$f_{pu}^{-1}(M,K)\tilde{\subseteq}sint_{\eta_{12}}[f_{pu}^{-1}(M,K)].$$

Therefore, $f_{pu}^{-1}(M,K)=sint_{\eta_{12}}[f_{pu}^{-1}(M,K)]$. Hence, $f_{pu}^{-1}(M,K)\in\eta_{12}$. This complete the proof.

Theorem 3.7. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let*

$$f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$$

be a soft bijection mapping. Then, f_{pu} is a p-soft continuous if and only if

$$sint_{\sigma_{12}}f_{pu}(G,E)\tilde{\subseteq}f_{pu}[sint_{\eta_{12}}(G,E)],\forall\ (G,E)\in SS(X)_E.$$

Proof. For each $(G,E)\in SS(X)_E$, we have $sint_{\sigma_{12}}f_{pu}(G,E)\tilde{\subseteq}f_{pu}(G,E)$. Therefore,

$$f_{pu}^{-1}[sint_{\sigma_{12}}f_{pu}(G,E)]\tilde{\subseteq}f_{pu}^{-1}f_{pu}(G,E).$$

Since f_{pu} is a soft injective, then $f_{pu}^{-1}[sint_{\sigma_{12}}f_{pu}(G,E)]\tilde{\subseteq}(G,E)$. Since $sint_{\sigma_{12}}f_{pu}(G,E)\in\sigma_{12}$ and f_{pu} is p-soft continuous, then $f_{pu}^{-1}[sint_{\sigma_{12}}f_{pu}(G,E)]\tilde{\subseteq}$
$sint_{\eta_{12}}(G,E)$. It follows that

$$sint_{\sigma_{12}}f_{pu}(G,E)\tilde{\subseteq}f_{pu}[sint_{\eta_{12}}(G,E)].$$

Conversely, let $(H,K)\in\sigma_{12}$. Then $f_{pu}^{-1}(H,K)\in SS(X)_E$ implies by hypothesis that

$$sint_{\sigma_{12}}f_{pu}[f_{pu}^{-1}(H,K)]\tilde{\subseteq}f_{pu}[sint_{\eta_{12}}f_{pu}^{-1}(H,K)].$$

But, f_{pu} is a soft surjective and $sint_{\sigma_{12}}(H,K)=(H,K)$, then $(H,K)\tilde{\subseteq}f_{pu}[sint_{\eta_{12}}f_{pu}^{-1}(H,K)]$. It follows that $f_{pu}^{-1}(H,K)\tilde{\subseteq}f_{pu}^{-1}f_{pu}[sint_{\eta_{12}}f_{pu}^{-1}(H,$
$K)]=sint_{\eta_{12}}f_{pu}^{-1}(H,K)$. Hence, $f_{pu}^{-1}(H,K)\tilde{\subseteq}sint_{\eta_{12}}f_{pu}^{-1}(H,K)$. Thus, $f_{pu}^{-1}(H,K)\in\eta_{12}$. Hence, f_{pu} is a p-soft continuous.

Theorem 3.8. *Let (X,η_1,η_2,E), (Y,σ_1,σ_2,K) and (Z,θ_1,θ_2,J) be three soft bitopological spaces.*

If $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ and $g_{p'u'}:(Y,\sigma_1,\sigma_2,K)\to(Z,\theta_1,\theta_2,J)$ are p-soft continuous mappings, then $g_{p'u'}of_{pu}:(X,\eta_1,\eta_2,E)\to(Z,\theta_1,\theta_2,J)$ is also p-soft continuous mapping.

Proof. Let $(D,J)\in\theta_{12}$. Since $g_{p'u'}:(Y,\sigma_1,\sigma_2,K)\to(Z,\theta_1,\theta_2,J)$ is a p-soft continuous, then $g_{p'u'}^{-1}(D,J)\in\sigma_{12}$. Also, since $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ is a p-soft continuous, then $f_{pu}^{-1}[g_{p'u'}^{-1}(D,J)]\in\eta_{12}$. But, $f_{pu}^{-1}[g_{p'u'}^{-1}(D,J)]=(g_{p'u'}of_{pu})^{-1}(D,J)$, then $(g_{p'u'}of_{pu})^{-1}(D,J)\in\eta_{12}$. Hence, $g_{p'u'}of_{pu}:(X,\eta_1,\eta_2,E)\to(Z,\theta_1,\theta_2,J)$ is p-soft continuous.

Definition 3.9. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let (x_e,E) be a soft point in $(\tilde{X},E)$. A soft mapping $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ is said to be a pairwise soft continuous at a soft point (x_e,E) if for each $(H,K)\in\sigma_{12}[f_{pu}(x_e,E)]$ there exists $(G,E)\in\eta_{12}[(x_e,E)]$ such that $f_{pu}(G,E)\tilde{\subseteq}(H,K)$, where*

$\sigma_{12}[f_{pu}(x_e,E)]=\{(H,K)\in\sigma_{12}:f_{pu}(x_e,E)\tilde{\subseteq}(H,K)\}$,
$\eta_{12}[(x_e,E)]=\{(G,E)\in\eta_{12}:(x_e,E)\tilde{\subseteq}(G,E)\}$.

Theorem 3.10. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let (x_e, E) be a soft point in $(\tilde{X}, E)$. Then, a soft mapping*

$f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ *is a p-soft continuous at (x_e, E) if and only if $f_{pu}^{-1}(H, K) \in \eta_{12}[(x_e, E)]$ for each $(H, K) \in \sigma_{12}[f_{pu}(x_e, E)]$.*

Proof. Immediate.

Theorem 3.11. *A soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous if and only if it is a p-soft continuous at each soft point in $(\tilde{X}, E)$.*

Proof. Let (x_e, E) be any soft point in $(\tilde{X}, E)$ and let $(H, K) \in \sigma_{12}[f_{pu}(x_e, E)]$. Then $(H, K) \in \sigma_{12}$ it follows by given that $f_{pu}^{-1}(H, K) \in \eta_{12}$. Now, since $f_{pu}(x_e, E)\tilde{\subseteq}(H, K)$, then $(x_e, E)\tilde{\subseteq}f_{pu}^{-1}(H, K)$. Therefore, $f_{pu}^{-1}(H, K) \in \eta_{12}[(x_e, E)]$. It follows by Theorem 3.10 that f_{pu} is a p-soft continuous at (x_e, E). Since (x_e, E) is an arbitrary soft point in $(\tilde{X}, E)$, then f_{pu} is a p-soft continuous at each soft point in $(\tilde{X}, E)$.

Conversely, let $(H, K) \in \sigma_{12}$. If $f_{pu}^{-1}(H, K) = (\tilde{\phi}, E)$, then $f_{pu}^{-1}(H, K) \in \eta_{12}$. If $f_{pu}^{-1}(H, K) \neq (\tilde{\phi}, E)$, then for each $(x_e, E)\tilde{\subseteq}f_{pu}^{-1}(H, K)$ we have $f_{pu}(x_e, E)\tilde{\subseteq}(H, K)$ and so $(H, K) \in \sigma_{12}[f_{pu}(x_e, E)]$. Consequently, by hypothesis, there exists $(G_{x_e}, E) \in \eta_{12}$ such that $(x_e, E)\tilde{\subseteq}(G_{x_e}, E)\tilde{\subseteq}f_{pu}^{-1}(H, K)$ for each $(x_e, E)\tilde{\subseteq}f_{pu}^{-1}(H, K)$. Clearly, $\tilde{\bigcup}\{(G_{x_e}, E) : (x_e, E)\tilde{\subseteq}f_{pu}^{-1}(H, K)\} = f_{pu}^{-1}(H, K)$. Hence, $f_{pu}^{-1}(H, K) \in \eta_{12}$. This complete the proof.

Theorem 3.12. *If $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous mapping, then $f_{pu} : (X, \eta_{p\Lambda}, E) \to (Y, \sigma_{p\Lambda}, K)$ is a soft continuous.*

Proof. Let (H, K) be any $p\Lambda$-open soft set in $(Y, \sigma_1, \sigma_2, K)$, i.e., $(H, K) \in \sigma_{p\Lambda}$. Then $sker_{\sigma_{12}}(H, K) = (H, K)$. It follows that $(H, K) = \tilde{\bigcap}\{(M, K) \in \sigma_{12} : (H, K)\tilde{\subseteq}(M, K)\}$. Since f_{pu} is p-soft continuous, then $f_{pu}^{-1}(M, K) \in \eta_{12}\ \forall\ (M, K) \in \sigma_{12}$. Therefore,
$f_{pu}^{-1}(H, K) = \tilde{\bigcap}\{f_{pu}^{-1}(M, K) \in \eta_{12} : (M, K) \in \sigma_{12}, f_{pu}^{-1}(H, K)\tilde{\subseteq}f_{pu}^{-1}(M, K)\}$. On the other hand, we have

$$\begin{aligned} sker_{\eta_{12}} f_{pu}^{-1}(H, K) &= \tilde{\bigcap}\{(G, E) \in \eta_{12} : f_{pu}^{-1}(H, K)\tilde{\subseteq}(G, E)\} \\ &\tilde{\subseteq}\tilde{\bigcap}\{f_{pu}^{-1}(M, K) \in \eta_{12} : (M, K) \in \sigma_{12}, f_{pu}^{-1}(H, K)\tilde{\subseteq}f_{pu}^{-1}(M, K)\} \\ &= f_{pu}^{-1}(H, K). \end{aligned}$$

So, $sker_{\eta_{12}} f_{pu}^{-1}(H, K)\tilde{\subseteq}f_{pu}^{-1}(H, K)$. But, $f_{pu}^{-1}(H, K)\tilde{\subseteq}sker_{\eta_{12}} f_{pu}^{-1}(H, K)$. It follows that $(H, K) \in \eta_{p\Lambda}$. Hence,

$f_{pu} : (X, \eta_{p\Lambda}, E) \to (Y, \sigma_{p\Lambda}, K)$ is a soft continuous mapping.

Definition 3.13. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. A soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is said to be:*

(1) *$p\Lambda$-soft continuous if $f_{pu}^{-1}(H, K) \in \eta_{p\Lambda}$ for any $(H, K) \in \sigma_{12}$.*

(2) *$p\lambda$-soft continuous if $f_{pu}^{-1}(H, K) \in \eta_{p\lambda}^c$ for any $(H, K) \in \sigma_{12}^c$.*

(3) *gp-soft continuous if $f_{pu}^{-1}(H, K) \in G_pC(X, \eta_1, \eta_2)_E$ for any $(H, K) \in \sigma_{12}^c$.*

(4) *pl-soft continuous if $f_{pu}^{-1}(H, K) \in PLCS(X, \eta_1, \eta_2)_E$ for any $(H, K) \in \sigma_{12}^c$.*

Theorem 3.14. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping. Let $\Gamma \in \{p\Lambda, p\lambda\}$. Then, the following statements are equivalent:*

(1) *f_{pu} is a Γ-soft continuous.*

(2) *$f_{pu}^{-1}(F, K) \in \eta_\Gamma^c$ for all $(F, K) \in \sigma_{12}^c$.*

(3) *$f_{pu}[scl_{\eta_\Gamma}(G, E)]\tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(G, E)]$ for any $(G, E) \in SS(X)_E$.*

(4) *$scl_{\eta_\Gamma}[f_{pu}^{-1}(H, K)]\tilde{\subseteq} f_{pu}^{-1}[scl_{\sigma_{12}}(H, K)]$ for any $(H, K) \in SS(Y)_K$.*

(5) *$f_{pu}^{-1}[sint_{\sigma_{12}}(H, K)]\tilde{\subseteq} sint_{\eta_\Gamma}[f_{pu}^{-1}(H, K)]$ for any $(H, K) \in SS(Y)_K$.*

Proof. It is similar to the proof of Theorem 3.6.

Theorem 3.15. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft bijection mapping. Then,*

f_{pu} is a Γ-soft continuous iff $sint_{\sigma_{12}}[f_{pu}(G, E)]\tilde{\subseteq} f_{pu}[sint_{\eta_\Gamma}(G, E)]$ $\forall$ $(G, E) \in SS(X)_E$, where $\Gamma \in \{p\Lambda, p\lambda\}$.

Proof. By similar to the proof of Theorem 3.7.

Theorem 3.16. *If $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a gp-soft continuous and (X, η_1, η_2, E) is a $PST^*_{\frac{1}{2}}$, then f_{pu} is a p-soft continuous.*

Proof. Let (H, K) be a p-closed soft set in $(Y, \sigma_1, \sigma_2, K)$. Then by given $f_{pu}^{-1}(H, K)$ is a gp-closed soft set in (X, η_1, η_2, E). It follows by $PST^*_{\frac{1}{2}}$ property that $f_{pu}^{-1}(H, K)$ is a p-closed soft set in (X, η_1, η_2, E). So, by Theorem 3.6, $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is a p-soft continuous.

Theorem 3.17. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping. Then,*

f_{pu} is a p-soft continuous iff it is $p\lambda$-soft continuous and gp-soft continuous.

Proof. The proof is direct by Theorem 2.30.

Lemma 3.18. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be soft injective mapping. Then,*

(1) *If $x^1_{e_1}, x^2_{e_2} \in \xi(X)_E$ such that $(x^1_{e_1}, E) \neq (x^2_{e_2}, E)$, then $f_{pu}(x^1_{e_1}, E) \neq f_{pu}(x^2_{e_2}, E)$.*

(2) *For every soft point (x_e, E) in $(\tilde{X}, E)$ there exists a soft point (y_k, K) in $(\tilde{Y}, K)$ such that $f_{pu}(x_e, E) = (y_k, K)$ and $(x_e, E) = f_{pu}^{-1}(y_k, K)$.*

Proof. Immediate from Definition 2.6.

Theorem 3.19. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a p-soft continuous and soft injective mapping.*

If $(Y, \sigma_1, \sigma_2, K)$ is a PST^*_i, then (X, η_1, η_2, E) is a PST^*_i, $i = 0, 1, 2$.

Proof. At $i = 0$, let $x^1_{e_1}, x^2_{e_2} \in \xi(X)_E$ such that $(x^1_{e_1}, E) \neq (x^2_{e_2}, E)$. Since f_{pu} is soft injective mapping, then by Lemma 3.18 there exist $y^1_{k_1}, y^2_{k_2} \in \xi(Y)_K$ such that $f_{pu}(x^1_{e_1}, E) = (y^1_{k_1}, K)$, $f_{pu}(x^2_{e_2}, E) = (y^2_{k_2}, K)$ and $(y^1_{k_1}, K) \neq (y^2_{k_2}, K)$. Then,

- If $(Y, \sigma_1, \sigma_2, K)$ is a PST_0^*, then, by Lemma 2.33, there exists $(H, K) \in \sigma_{12} \bigcup \sigma_{12}^c$ such that $f_{pu}(x^1_{e_1}, E)\tilde{\subseteq}$ (H, K), $f_{pu}(x^2_{e_2}, E)\tilde{\subseteq}(H, K)^c$. It follows that $(x^1_{e_1}, E)\tilde{\subseteq} f^{-1}_{pu}(H, K)$, $(x^2_{e_2}, E)\tilde{\subseteq} f^{-1}_{pu}(H, K)^c$ $= [f^{-1}_{pu}(H, K)]^c$. Since f_{pu} is a p-soft continuous, then $f^{-1}_{pu}(H, K) \in \eta_{12} \bigcup \eta_{12}^c$. It follows by Lemma 2.33 that (X, η_1, η_2, E) is a PST_0^*.
- If $(Y, \sigma_1, \sigma_2, K)$ is a PST_1^*, then by similar way we can easily prove that (X, η_1, η_2, E) is a PST_1^*.
- If $(Y, \sigma_1, \sigma_2, K)$ is a PST_2^*, there exists $(H, K), (M, K) \in \sigma_{12}$ such that $(y^1_{k_1}, K)\tilde{\subseteq}(H, K)$, $(y^2_{k_2}, K)\tilde{\subseteq}(M, K)$ and $(H, K)\tilde{\cap}(M, K) = (\tilde{\phi}, K)$. It follows that $(x^1_{e_1}, E)\tilde{\subseteq} f^{-1}_{pu}(H, K)$, $(x^2_{e_2}, E)\tilde{\subseteq} f^{-1}_{pu}(M, K)$ and $f^{-1}_{pu}[(H, K)$ $\tilde{\cap}(M, K)] = f^{-1}_{pu}(H, K) \,\tilde{\cap} f^{-1}_{pu}(M, K) = (\tilde{\phi}, E)$. Since f_{pu} is a p-soft continuous, then $f^{-1}_{pu}(H, K), f^{-1}_{pu}(M,$ $K) \in \eta_{12}$. Hence, (X, η_1, η_2, E) is a PST_2^*.

Corollary 3.20. *From Theorems 2.22, 2.20, 2.25 and 2.29 we can deduce the following implications:*

$$
\begin{array}{ccccc}
gp\text{-soft continuous} & \Leftarrow & p\text{-soft continuous} & \Rightarrow & p\Lambda\text{-soft continuous} \\
 & & \Downarrow & & \Downarrow \\
 & & pl\text{-soft continuous} & \Rightarrow & p\lambda\text{-soft continuous}
\end{array}
$$

4 Pairwise soft open, soft closed and soft homeomorphism mappings

Definition 4.1. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. A soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is said to be a pairwise soft open mapping [briefly, p-soft open] if the image of any p-open soft set in (X, η_1, η_2, E) is a p-open soft set in $(Y, \sigma_1, \sigma_2, K)$, i.e., $f_{pu}(G, E) \in \sigma_{12}$ for any $(G, E) \in \eta_{12}$.*

Example 4.2. *Let $X = \{x, y, z, w\}$, $Y = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $K = \{k_1, k_2, k_3\}$. Let*

$\eta_1 = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E)\}$,

$\eta_2 = \{(\tilde{\phi}, E), (\tilde{X}, E), (H, E)\}$,

where

$(G, E) = \{(e_1, \{x\}), (e_2, \{y, z\}), (e_3, \{x, y\})\}$,

$(H, E) = \{(e_1, \{x, y\}), (e_2, \{y, z\}), (e_3, \{x, y, w\})\}$,

and let

$\sigma_1 = \{(\tilde{\phi}, K), (\tilde{Y}, K), (M, K), (N, K)\}$,

$\sigma_2 = \{(\tilde{\phi}, K), (\tilde{X}, K), (F, K)\}$,

where

$(M, K) = \{(k_1, \{a, c\}), (k_2, \{a, b\}), (k_3, \{c\})\}$,

$(N, K) = \{(k_1, \{c\}), (k_2, \{a, b\}), (k_3, \{c\})\}$,

$(F, K) = \{(k_1, \{a, c\}), (k_2, \{a, b\}), (k_3, \{a, c\})\}$.

Then (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ are sbts.

Moreover,

$\eta_{12} = \{(\tilde{\phi}, E), (\tilde{X}, E), (G, E), (H, E)\}$ *and*

$\sigma_{12} = \{(\tilde{\phi}, K), (\tilde{Y}, K), (F, K), (M, K), (N, K)\}$.

Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping defined by

$u(x) = u(w) = \{c\}, u(y) = \{a\}, u(z) = \{b\}$, *and*

$p(e_1) = \{k_3\}, p(e_2) = \{k_2\}, p(e_3) = \{k_1\}$.

It is easy to prove that

$f_{pu}(x_{e_1}, E) = (c_{k_3}, K),\ f_{pu}(x_{e_2}, E) = (c_{k_2}, K),\ f_{pu}(x_{e_3}, E) = (c_{k_1}, K)$,

$f_{pu}(y_{e_1}, E) = (a_{k_3}, K),\ f_{pu}(y_{e_2}, E) = (a_{k_2}, K),\ f_{pu}(y_{e_3}, E) = (a_{k_1}, K)$,

$f_{pu}(z_{e_1}, E) = (b_{k_3}, K),\ f_{pu}(z_{e_2}, E) = (b_{k_2}, K),\ f_{pu}(z_{e_3}, E) = (b_{k_1}, K)$,

$f_{pu}(w_{e_1}, E) = (c_{k_3}, K),\ f_{pu}(w_{e_2}, E) = (c_{k_2}, K),\ f_{pu}(w_{e_3}, E) = (c_{k_1}, K)$.

Furthermore, we can prove that

$f_{pu}(G, E) = (M, K)$, $f_{pu}(H, E) = (F, K)$, $f_{pu}(\tilde{X}, E) = (\tilde{Y}, K)$, *and* $f_{pu}(\tilde{\phi}, E) = (\tilde{\phi}, K)$. *Hence f_{pu} is a p- soft open mapping. It is clear that f_{pu} is not p-soft continuous because $(N, K) \in \sigma_{12}$ but $f_{pu}^{-1}(N, K) = \{(e_1, \{x, w\}), (e_2, \{y, z\}), (e_3, \{x, w\})\} \notin \eta_{12}$.*

Theorem 4.3. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let $f_{pu} : (X, \eta_1, \eta_2, E)$ $\to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping. Then, the following statements are equivalent:*

(1) *f_{pu} is a p-soft open mapping.*

(2) *$f_{pu}[sint_{\eta_{12}}(G, E)] \tilde{\subseteq} sint_{\sigma_{12}}[f_{pu}(G, E)]$ for any $(G, E) \in SS(X)_E$.*

(3) *$sint_{\eta_{12}}[f_{pu}^{-1}(H, K)] \tilde{\subseteq} f_{pu}^{-1}[sint_{\sigma_{12}}(H, K)]$ for any $(H, K) \in SS(Y)_K$.*

(4) *$\forall\ (x_e, E) \in \xi(X)_E$, $(G, E) \in \eta_{12}[(x_e, E)] \Rightarrow f_{pu}(G, E) \in \sigma_{12}[f_{pu}(x_e, E)]$.*

(5) *$f_{pu}^{-1}[scl_{\sigma_{12}}(H, K)] \tilde{\subseteq} scl_{\eta_{12}}[f_{pu}^{-1}(H, K)]$ for any $(H, K) \in SS(Y)_K$.*

Proof. (1) $\Rightarrow$ (2): Let $(G, E) \in SS(X)_E$. Then, $sint_{\sigma_{12}}[f_{pu}(G, E)] \tilde{\subseteq} f_{pu}(G, E)$. Since $sint_{\eta_{12}}(G, E) \in \eta_{12}$, then by (1) we deduce that $f_{pu}[sint_{\eta_{12}}(G, E)] \in \sigma_{12}$ and $f_{pu}[sint_{\eta_{12}}(G, E)] \tilde{\subseteq} f_{pu}(G, E)$. But, $sint_{\sigma_{12}}[f_{pu}(G, E)]$ is the largest p-open soft set in $(Y, \sigma_1, \sigma_2, K)$ containing $f_{pu}(G, E)$, then $f_{pu}[sint_{\eta_{12}}(G, E)]$ $\tilde{\subseteq} sint_{\sigma_{12}}[f_{pu}(G, E)]$. Hence, (2) holds.

(2) $\Rightarrow$ (3): Let $(H, K) \in SS(Y)_K$. Certain that $f_{pu}^{-1}(H, K) \in SS(X)_E$. It follows by (2) that $f_{pu}[sint_{\eta_{12}} f_{pu}^{-1}(H, K)] \tilde{\subseteq} sint_{\sigma_{12}}[f_{pu} f_{pu}^{-1}(H, K)]$. But, $f_{pu} f_{pu}^{-1}(H, K) \tilde{\subseteq} (H, K)$ and so $sint_{\sigma_{12}}[f_{pu} f_{pu}^{-1}(H, K)] \tilde{\subseteq}$ $sint_{\sigma_{12}}(H, K)$. Therefore, $f_{pu}[sint_{\eta_{12}} f_{pu}^{-1}(H, K)] \tilde{\subseteq} sint_{\sigma_{12}}(H, K)$ which implies that $sint_{\eta_{12}} f_{pu}^{-1}(H, K) \tilde{\subseteq} f_{pu}^{-1}$ $[sint_{\sigma_{12}}(H, K)]$. Hence, (3) holds.

(3) $\Rightarrow$ (4): Let (x_e, E) be a soft set in $(\tilde{X}, E)$ and let $(G, E) \in \eta_{12}[(x_e, E)]$. Then $(x_e, E) \tilde{\subseteq} (G, E)$ which implies that $f_{pu}(x_e, E) \tilde{\subseteq} f_{pu}(G, E)$. It remains prove that $f_{pu}(G, E)$ is a p-open soft set in

$(Y, \sigma_1, \sigma_2, K)$. Since $f_{pu}(G, E) \in SS(Y)_K$, then by (3) we have $sint_{\eta_{12}}[f_{pu}^{-1} f_{pu}(G, E)] \tilde{\subseteq} f_{pu}^{-1}[sint_{\sigma_{12}} f_{pu}(G, E)]$. It follows that $sint_{\eta_{12}}(G, E) \tilde{\subseteq} f_{pu}^{-1}[sint_{\sigma_{12}} f_{pu}(G, E)]$, implies $f_{pu}[sint_{\eta_{12}}(G, E)] \tilde{\subseteq} f_{pu} f_{pu}^{-1}[sint_{\sigma_{12}} f_{pu}(G, E)]$. But $sint_{\eta_{12}}(G, E) = (G, E)$, then $f_{pu}(G, E) \tilde{\subseteq} sint_{\sigma_{12}} f_{pu}(G, E)$. Thus, $f_{pu}(G, E) \in \sigma_{12}$ and containing $f_{pu}(x_e, E)$. Hence, (4) holds.

(4) $\Rightarrow$ (5): Let $(H, K) \in SS(Y)_K$ and let $(x_e, E) \tilde{\in} f_{pu}^{-1}[scl_{\sigma_{12}}(H, K)]$. Then, $f_{pu}(x_e, E) \tilde{\in} scl_{\sigma_{12}}(H, K)$. Now, for all $(M, E) \in \eta_{12}[(x_e, E)]$ we have by (4) $f_{pu}(M, E) \in \sigma_{12}[f_{pu}(x_e, E)]$ implies $f_{pu}(x_e, E) \tilde{\in} f_{pu}(M, E)$, $f_{pu}(M, E) \in \sigma_{12}$. It follows that $f_{pu}(M, E) \tilde{\cap} (H, K) \neq (\tilde{\phi}, K)$. Thus there exists $(m_\alpha, E) \tilde{\in} (M, E)$ such that $f_{pu}(m_\alpha, E) \tilde{\in} (H, K)$. So, $(m_\alpha, E) \tilde{\in} f_{pu}^{-1}(H, K)$ which implies that $(M, E) \tilde{\cap} f_{pu}^{-1}(H, K) \neq (\tilde{\phi}, E)$ all $(M, E) \in \eta_{12}[(x_e, E)]$. Hence, $(x_e, E) \tilde{\in} scl_{\eta_{12}}[f_{pu}^{-1}(H, K)]$. Thus, (5) holds.

(5) $\Rightarrow$ (2): From properties of p-soft closure and p-soft interior operators we have

$sint_{\eta_{12}}(G, E) = (\tilde{X}, E) \setminus [scl_{\eta_{12}}[(\tilde{X}, E) \setminus (G, E)]]$. So,

$$\begin{aligned} sint_{\eta_{12}}[f_{pu}^{-1} f_{pu}(G, E)] &= (\tilde{X}, E) \setminus [scl_{\eta_{12}}[(\tilde{X}, E) \setminus f_{pu}^{-1} f_{pu}(G, E)]] \\ &= (\tilde{X}, E) \setminus scl_{\eta_{12}}[f_{pu}^{-1}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]] \text{ [by Theorem 2.10 (1)]} \\ &\tilde{\subseteq} (\tilde{X}, E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]] \text{ [by (5)]} \end{aligned}$$

Hence,

$$sint_{\eta_{12}}[f_{pu}^{-1} f_{pu}(G, E)] \tilde{\subseteq} (\tilde{X}, E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]] ......(*).$$

Now, since $(G, E) \tilde{\subseteq} f_{pu}^{-1} f_{pu}(G, E)$, then $sint_{\eta_{12}}(G, E) \tilde{\subseteq} sint_{\eta_{12}}[f_{pu}^{-1} f_{pu}(G, E)]$. Thus, by (*) we deduce that

$sint_{\eta_{12}}(G, E) \tilde{\subseteq} (\tilde{X}, E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]]$ which implies that

$$\begin{aligned} f_{pu}[sint_{\eta_{12}}(G, E)] &\tilde{\subseteq} f_{pu}[(\tilde{X}, E) \setminus f_{pu}^{-1}[scl_{\sigma_{12}}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]] \\ &= f_{pu}[f_{pu}^{-1}[(\tilde{Y}, K) \setminus [scl_{\sigma_{12}}[(\tilde{Y}, K) \setminus f_{pu}(G, E)]] \text{ [by Theorem 2.10 (1)]} \\ &= f_{pu}[f_{pu}^{-1}[sint_{\sigma_{12}} f_{pu}(G, E)] \\ &\tilde{\subseteq} sint_{\sigma_{12}} f_{pu}(G, E). \text{ Hence, (2) holds.} \end{aligned}$$

(2) $\Rightarrow$ (1): It is obvious.

Definition 4.4. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. A soft mapping $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ is said to be a pairwise soft closed mapping [briefly, p-soft closed mapping] if the image of any p-closed soft set in (X, η_1, η_2, E) is a p-closed soft set in $(Y, \sigma_1, \sigma_2, K)$, i.e., $f_{pu}(F, E) \in \sigma_{12}^c$ for any $(F, E) \in \eta_{12}^c$.*

Theorem 4.5. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces and let $f_{pu} : (X, \eta_1, \eta_2, E)$ $\to (Y, \sigma_1, \sigma_2, K)$ be a soft mapping. Then, f_{pu} is a p-soft closed mapping iff $scl_{\sigma_{12}}[f_{pu}(G, E)] \tilde{\subseteq} f_{pu}[scl_{\eta_{12}}(G, E)]$ for any $(G, E) \in SS(X)_E$.*

Proof. Necessity: For any soft set (G, E) over X we have $scl_{\sigma_{12}}[f_{pu}(G, E)] \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}[scl_{\eta_{12}}(G, E)]]$. Since f_{pu} is a p-soft closed mapping and $scl_{\eta_{12}}(G, E) \in \eta_{12}^c$, then $f_{pu}[scl_{\eta_{12}}(G, E)] \in \sigma_{12}^c$, i.e., $scl_{\sigma_{12}}[f_{pu}[scl_{\eta_{12}}$ $(G, E)]] = f_{pu}[scl_{\eta_{12}}(G, E)]$. Therefore, $scl_{\sigma_{12}}[f_{pu}(G, E)] \tilde{\subseteq} f_{pu}[scl_{\eta_{12}}(G, E)]]$.

Sufficiency: Let $(G,E) \in \eta_{12}^c$. Then $scl_{\eta_{12}}(G,E) = (G,E)$. It follows by given that $scl_{\sigma_{12}}[f_{pu}(G,E)] \tilde{\subseteq} f_{pu}(G,E)$. Hence, $f_{pu}(G,E) = scl_{\sigma_{12}}[f_{pu}(G,E)]$. Thus, $f_{pu}(G,E) \in \sigma_{12}^c$. Hence, f_{pu} is p-soft closed mapping.

Lemma 4.6. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let $f_{pu} : (X,\eta_1,\eta_2,E) \to (Y,\sigma_1,\sigma_2,K)$ be a soft mapping. Then $f_{pu}^{-1}(H,K) \tilde{\subseteq} (M,E)$ iff $(H,K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}[(\tilde{X},E) \setminus (M,E)]$.*

Proof. Straightforward.

Theorem 4.7. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let $f_{pu} : (X,\eta_1,\eta_2,E) \to (Y,\sigma_1,\sigma_2,K)$ be a soft mapping. Then f_{pu} is a p-soft closed mapping iff $\forall$ $(y_k,K) \in \xi(Y)_K$, $\forall$ $(G,E) \in \eta_{12}$ with $f_{pu}^{-1}(y_k,K) \tilde{\subseteq} (G,E)$, there exists $(H,K) \in \sigma_{12}[(y_k,K)]$ such that $f_{pu}^{-1}(H,K) \tilde{\subseteq} (G,E)$.*

Proof. Let f_{pu} be a p-soft closed mapping. Let $(y_k,K) \in \xi(Y)_K$ and $(G,E) \in \eta_{12}$ such that $f_{pu}^{-1}(y_k,K) \tilde{\subseteq} (G,E)$. Then by Lemma 4.6 we have $(y_k,K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}[(\tilde{X},E) \setminus (G,E)]$. Since $(\tilde{X},E) \setminus (G,E) \in \eta_{12}^c$, then by given $f_{pu}[(\tilde{X},E) \setminus (G,E)] \in \sigma_{12}^c$. So, $(\tilde{Y},K) \setminus f_{pu}[(\tilde{X},E) \setminus (G,E)] \in \sigma_{12}[(y_k,K)]$. Take $(\tilde{Y},K) \setminus f_{pu}[(\tilde{X},E) \setminus (G,E)] = (H,K)$. It is easy to verify that $f_{pu}^{-1}(H,K) \tilde{\subseteq} (G,E)$.

Conversely, let $(F,E) \in \eta_{12}^c$. We shall prove that $f_{pu}(F,E) \in \sigma_{12}^c$ or equivalently, $(\tilde{Y},K) \setminus f_{pu}(F,E) \in \sigma_{12}$. Let $(y_k,K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}(F,E)$. Then

$$\begin{aligned} f_{pu}^{-1}(y_k,K) &\tilde{\subseteq} f_{pu}^{-1}[(\tilde{Y},K) \setminus f_{pu}(F,E)] \\ &= (\tilde{X},E) \setminus f_{pu}^{-1} f_{pu}(F,E) \\ &\tilde{\subseteq} (\tilde{X},E) \setminus (F,E). \end{aligned}$$

Hence, $f_{pu}^{-1}(y_k,K) \tilde{\subseteq} (\tilde{X},E) \setminus (F,E)$. Therefore, by given there exists $(H_{y_k},K) \in \sigma_{12}[(y_k,K)]$ such that $f_{pu}^{-1}(H_{y_k},K) \tilde{\subseteq} (\tilde{X},E) \setminus (F,E)$. Hence, $(H_{y_k},K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}(F,E)$ [by Lemma 4.6]. Consequently, for every $(y_k,K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}(F,E)$ there exists $(H_{y_k},K) \in \sigma_{12}$ such that $(y_k,K) \tilde{\subseteq} (H_{y_k},K) \tilde{\subseteq} (\tilde{Y},K) \setminus f_{pu}(F,E)$. It is clear that $\tilde{\bigcup}(H_{y_k},K) = (\tilde{Y},K) \setminus f_{pu}(F,E)$. It follows that $(\tilde{Y},K) \setminus f_{pu}(F,E) \in \sigma_{12}$. Hence, $f_{pu}(F,E) \in \sigma_{12}^c$. This complete the proof.

Definition 4.8. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces. A soft mapping $f_{pu} : (X,\eta_1,\eta_2,E) \to (Y,\sigma_1,\sigma_2,K)$ is said to be a pairwise soft homeomorphism mapping [briefly, p-soft hom] if f_{pu} is a soft bijection, p-soft continuous and p-soft open mapping.*

Theorem 4.9. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let $f_{pu} : (X,\eta_1,\eta_2,E) \to (Y,\sigma_1,\sigma_2,K)$ be a soft mapping. Then, the following statements are equivalent:*

(1) *f_{pu} is a p-soft homeomorphism mapping.*

(2) *$f_{pu}[sint_{\eta_{12}}(G,E)] = sint_{\sigma_{12}}[f_{pu}(G,E)]$ for any $(G,E) \in SS(X)_E$.*

(3) *$sint_{\eta_{12}}[f_{pu}^{-1}(H,K)] = f_{pu}^{-1}[sint_{\sigma_{12}}(H,K)]$ for any $(H,K) \in SS(Y)_K$.*

(4) *$f_{pu}^{-1}[scl_{\sigma_{12}}(H,K)] = scl_{\eta_{12}}[f_{pu}^{-1}(H,K)]$ for any $(H,K) \in SS(Y)_K$.*

(5) *$f_{pu}[scl_{\eta_{12}}(G,E)] = scl_{\sigma_{12}}[f_{pu}(G,E)]$ for any $(G,E) \in SS(X)_E$.*

Proof. Immediate.

Definition 4.10. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces. A soft mapping $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ is said to be:*

(1) *$p\Lambda$-soft open(closed) mapping if $f_{pu}(F,E)\in\sigma_{p\Lambda}(\sigma^c_{p\Lambda})$ for any $(F,E)\in\eta_{12}(\eta^c_{12})$, respectively.*

(2) *$p\lambda$-soft open(closed) mapping if $f_{pu}(F,E)\in\sigma_{p\lambda}(\sigma^c_{p\lambda})$ for any $(F,E)\in\eta_{12}(\eta^c_{12})$, respectively.*

(3) *gp-soft open (closed) mapping if $f_{pu}(F,E)\in G_pO(Y,\sigma_1,\sigma_2)_E(G_pC(Y,\sigma_1,\sigma_2)_E)$ for any $(F,E)\in\eta_{12}(\eta^c_{12})$, respectively.*

(4) *pl-soft open(closed) mapping if $f_{pu}(F,E)\in PLOS(Y,\sigma_1,\sigma_2)_E(PLCS(Y,\sigma_1,\sigma_2)_E)$ for any $(F,E)\in\eta_{12}(\eta^c_{12})$, respectively.*

Theorem 4.11. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ be a soft mapping. Let $\Gamma\in\{p\Lambda,p\lambda\}$. Then, the following statements are equivalent:*

(1) *f_{pu} is a Γ-soft open mapping.*

(2) *$f_{pu}[sint_{\eta_{12}}(G,E)]\tilde{\subseteq}sint_{\sigma_\Gamma}[f_{pu}(G,E)]$ for any $(G,E)\in SS(X)_E$.*

(3) *$sint_{\eta_{12}}[f^{-1}_{pu}(H,K)]\tilde{\subseteq}f^{-1}_{pu}[sint_{\sigma_\Gamma}(H,K)]$ for any $(H,K)\in SS(Y)_K$*

(4) *$f^{-1}_{pu}[scl_{\sigma_\Gamma}(H,K)]\tilde{\subseteq}scl_{\eta_{12}}[f^{-1}_{pu}(H,K)]$ for any $(H,K)\in SS(Y)_K$.*

Proof. It is similar to the proof of Theorem 4.3.

Theorem 4.12. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces and let $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ be a soft mapping. Then,*

f_{pu} is a $p\Lambda$-soft closed mapping iff $scl_{\sigma_\Gamma}[f_{pu}(G,E)]\tilde{\subseteq}f_{pu}[scl_{\eta_{12}}(G,E)]$ for any $(G,E)\in SS(X)_E$, where $\Gamma\in\{p\Lambda,p\lambda\}$.

Proof. Straightforward.

Corollary 4.13. *The following implications are holds:*

$$\begin{array}{ccccc}
\textit{gp-soft open mapping} & \Leftarrow & \textit{p-soft open mapping} & \Rightarrow & p\Lambda\textit{-soft open mapping} \\
 & & \Downarrow & & \Downarrow \\
 & & \textit{pl-soft open mapping} & \Rightarrow & p\lambda\textit{-soft open mapping}
\end{array}$$

Corollary 4.14. *The following implications are holds:*

$$\begin{array}{ccccc}
\textit{gp-soft closed mapping} & \Leftarrow & \textit{p-soft closed mapping} & \Rightarrow & p\Lambda\textit{-soft closed mapping} \\
 & & \Downarrow & & \Downarrow \\
 & & \textit{pl-soft closed mapping} & \Rightarrow & p\lambda\textit{-soft closed mapping}
\end{array}$$

Theorem 4.15. *Let (X,η_1,η_2,E) and (Y,σ_1,σ_2,K) be two soft bitopological spaces. Let $f_{pu}:(X,\eta_1,\eta_2,E)\to(Y,\sigma_1,\sigma_2,K)$ be a p-soft continuous, p-soft open and soft injective mapping.*

If (Y,σ_1,σ_2,K) is a PSR^*_0, then (X,η_1,η_2,E) is a PSR^*_0.

Proof. Let $(x^1_{e_1}, E)$ and $(x^2_{e_2}, E)$ be two soft point in $\tilde{X}$ such that $x^1_{e_1} \tilde{\in} scl_{\eta_{12}}(x^2_{e_2}, E)$. Then, by Theorem 3.6 (3), $f_{pu}(x^1_{e_1}, E) \tilde{\subseteq} f_{pu}[scl_{\eta_{12}}(x^2_{e_2}, E)] \tilde{\subseteq} scl_{\sigma_{12}}[f_{pu}(x^2_{e_2}, E)]$. Since $x^1_{e_1}, x^2_{e_2} \in \xi(X)_E$, then by Lemma 3.18 (2) there exist $y^1_{k_1}$, $y^2_{k_2} \in \xi(Y)_K$ such that $f_{pu}(x^1_{e_1}, E) = (y^1_{k_1}, K)$, $f_{pu}(x^2_{e_2}, E) = (y^2_{k_2}, K)$. Therefore, $(y^1_{k_1}, K) \tilde{\subseteq} scl_{\sigma_{12}}(y^2_{k_2}, K)$. It follows by given that

$$\begin{aligned}(y^2_{k_2}, K) \tilde{\subseteq} scl_{\sigma_{12}}(y^1_{k_1}, K) &\Rightarrow f^{-1}_{pu}(y^2_{k_2}, K) \tilde{\subseteq} f^{-1}_{pu}[scl_{\sigma_{12}}(y^1_{k_1}, K)]\\ &\Rightarrow f^{-1}_{pu}(y^2_{k_2}, K) \tilde{\subseteq} scl_{\eta_{12}}[f^{-1}_{pu}(y^1_{k_1}, K)] \text{ [by Theorem 4.3 (5)]}\\ &\Rightarrow (x^2_{e_2}, E) \tilde{\subseteq} scl_{\eta_{12}}(x^1_{e_1}, E) \text{ [by Lemma 3.18 (2)].}\end{aligned}$$

Consequently, (X, η_1, η_2, E) is a PSR^*_0.

Lemma 4.16. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a soft surjective mapping. Then,*

if $y^1_{k_1}, y^2_{k_2} \in \xi(Y)_K$ such that $(y^1_{k_1}, K) \neq (y^2_{k_2}, K)$, then there exist $x^1_{e_{k_1}}, x^2_{e_{k_2}} \in \xi(X)_E$ such that $(x^1_{e_{k_1}}, E) \neq (x^2_{e_{k_2}}, E)$ and $f_{pu}(x^1_{e_{k_1}}, E) = (y^1_{k_1}, K)$, $f_{pu}(x^2_{e_{k_2}}, E) = (y^2_{k_2}, K)$.

Proof. Let $y^1_{k_1}, y^2_{k_2} \in \xi(Y)_K$ such that $(y^1_{k_1}, K) \neq (y^2_{k_2}, K)$. Then either $y^1 \neq y^2$ or $k_1 \neq k_2$.

- **Case (1):** Let $y^1 \neq y^2$. Since u is a soft surjective mapping, then there exist $x^1, x^2 \in X$ such that $u(x^1) = y^1$, $u(x^2) = y^2$ and $x^1 \neq x^2$. Since p is a surjective mapping, then there exist $e_{k_1}, e_{k_2} \in E$ such that $p(e_{k_1}) = k_1$, $p(e_{k_2}) = k_2$. Therefore, $x^1_{e_{k_1}} \neq x^2_{e_{k_2}}$ implies that $(x^1_{e_{k_1}}, E) \neq (x^2_{e_{k_2}}, E)$. Now, $f_{pu}(x^1_{e_{k_1}}, E)(k_1) = \bigcup_{e^* \in p^{-1}(k_1)} u[x^1_{e_{k_1}}(e^*)] = u(x^1) = y^1$, $f_{pu}(x^1_{e_{k_1}}, E)(k_i) = \bigcup_{\alpha^* \in p^{-1}(k_2)} u[x^1_{e_{k_1}}(\alpha^*)] = \phi$, $\forall\ k_i \in K\ , k_i \neq k_1$. Hence, $f_{pu}(x^1_{e_{k_1}}, E) = (y^1_{k_1}, K)$. By similar way we can deduce that $f_{pu}(x^2_{e_{k_2}}, E) = (y^2_{k_2}, K)$.

- **Case (2):** Let $k_1 \neq k_2$. Since p is a soft surjective mapping, then there exist $e_{k_1}, e_{k_2} \in E$ such that $p(e_{k_1}) = k_1$, $p(e_{k_2}) = k_2$ and $e_{k_1} \neq e_{k_2}$. Since $y^1, y^2 \in Y$ and u is a surjective mapping, then there exist $x^1, x^2 \in X$ such that $u(x^1) = y^1$ and $u(x^2) = y^2$. Now, since $e_{k_1} \neq e_{k_2}$, then $(x^1_{e_{k_1}}, E) \neq (x^2_{e_{k_2}}, E)$. Now, $f_{pu}(x^1_{e_{k_1}}, E)(k_1) = \bigcup_{e^* \in p^{-1}(k_1)} u[x^1_{e_{k_1}}(e^*)] = u(x^1) = y^1$ and $f_{pu}(x^1_{e_{k_1}}, E)(k_i) = \bigcup_{e^* \in p^{-1}(k_i)} u[x^1_{e_{k_1}}(e^*)] = \phi$, $\forall\ k_i \in K, k_i \neq k_1$ [because $e_{k_1} \notin p^{-1}(k_i)$]. Hence, $f_{pu}(x^1_{e_{k_1}}, E) = (y^1_{k_1}, K)$. By similar way we can deduce that $f_{pu}(x^2_{e_{k_2}}, E) = (y^2_{k_2}, K)$. This complete the proof.

Theorem 4.17. *Let (X, η_1, η_2, E) and $(Y, \sigma_1, \sigma_2, K)$ be two soft bitopological spaces. Let $f_{pu} : (X, \eta_1, \eta_2, E) \to (Y, \sigma_1, \sigma_2, K)$ be a p-soft open and soft surjective mapping.*

If (X, η_1, η_2, E) is a PST^*_i, then $(Y, \sigma_1, \sigma_2, K)$ is a PST^*_i, $i = 0, 1, 2$.

Proof. At $i = 0$. Let $y^1_{k_1}, y^2_{k_2} \in \xi(Y)_K$ such that $(y^1_{k_1}, K) \neq (y^2_{k_2}, K)$. Then, by Lemma 4.16, there exist $x^1_{e_{k_1}}, x^2_{e_{k_2}} \in \xi(X)_E$ such that $f_{pu}(x^1_{e_{k_1}}, E) = (y^1_{k_1}, K)$ and $f_{pu}(x^2_{e_{k_2}}, E) = (y^2_{k_2}, K)$. It follows by given that there exists $(G, E) \in \eta_{12} \bigcup \eta^c_{12}$ such that $(x^1_{e_{k_1}}, E) \tilde{\in} (G, E)$ and $(x^2_{e_{k_2}}, E) \tilde{\in} (G, E)^c$ or equivalently, $(x^1_{e_{k_1}}, E) \tilde{\subseteq} (G, E)$ and $(x^2_{e_{k_2}}, E) \tilde{\subseteq} (G, E)^c$ implies $f_{pu}(x^1_{e_{k_1}}, E) \tilde{\subseteq} f_{pu}(G, E)$ and $f_{pu}(x^2_{e_{k_2}}, E) \tilde{\subseteq} f_{pu}(G, E)^c$. So, $(y^1_{k_1}, K) \tilde{\subseteq} f_{pu}(G, E)$ and $(y^2_{k_2}, K) \tilde{\subseteq} f_{pu}(G, E)^c$. Now, if $(G, E) \in \eta_{12}$, then $f_{pu}(G, E) \in \sigma_{12}$ and if $(G, E) \in \eta^c_{12}$, then $f_{pu}(G, E) = f_{pu}(G, E) \in \sigma^c_{12}$. Therefore, there exists $(H, K) = f_{pu}(G, E) \in \sigma_{12} \bigcup \sigma^c_{12}$ such that $(y^1_{k_1}, K) \tilde{\subseteq} f_{pu}(G, E)$ and $(y^1_{k_1}, K) \tilde{\subseteq} [f_{pu}(G, E)]^c$. Consequently, $(Y, \sigma_1, \sigma_2, K)$ is a PST^*_0.

The proof at $i = 1, 2$ by similar way.

5 Conclusion

After 1999, many researchers worked on the findings of entities of soft sets theory introduced by Molodtsov and applied to many problems having uncertainties. In present paper, we introduced some new forms of soft continuity namely, pairwise soft continuous, pairwise Λ soft continuous, pairwise λ soft continuous, pairwise locally soft continuous and generalized pairwise soft continuous mappings in soft bitopological spaces. Furthermore, we also offered the notions of pairwise soft open mapping, pairwise soft open mapping and pairwise soft homeomorphism mapping and discussed many of their characterizations, properties and some relationships between them . It is necessary to continue more research to upgrade the general framework and to apply for the practical life applications.

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