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Some degree based connectivity indices of Kulli cycle Windmill graph

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Abstract The Kulli cycle windmill graph $C_{n+1}^{(m)}$ is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geqslant 3$ with a vertex K_1 in common. In this paper, we compute Zagreb, hyper-Zagreb, Randic connectivity, general Randic connectivity, sum connectivity, general sum connectivity, atom-bond connectivity and geometric-arithmetic indices of Kulli cycle windmill graph.

Key Words Zagreb index, hyper-Zagreb index, Randic connectivity index, sum connectivity index, ABC index

MSC 2010 05C05, 05C07

1 Introduction

In this paper, we consider only finite, connected, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [6].

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecular is called a topological index for that graph. There are numerous molecular descriptors, which are also referred to as topological descriptors that have found some applications in theoretical chemistry, especially in QSPR/QSAR research. In [5], the first and second Zagreb indices were introduced to take account of the contributions of pairs of adjacent vertices.

The first and second Zagreb indices of a graph G are defined as $M_1(G) = \sum_{v\in V(G)} d_G(v)^2$ or $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$.

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The first hyper Zagreb index of a graph G are defined as $HM_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$. This topological index was defined by Shirdel et al., in [9].

The second hyper Zagreb index of a graph G are defined as $HM_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$. This topological index was proposed by Farahani et al., in [3].

The Randic index or product connectivity index of a graph G is defined as $\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$. This index was proposed by Randic in [8].

The sum connectivity index of a graph G is defined as $X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$. This topological index was proposed by Zhou and Trinajstic in [11].

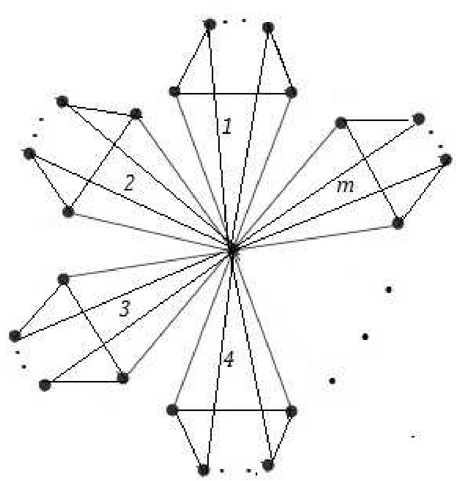
The general Randic connectivity index or second K_a index of a graph G is defined as $\chi^a(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]^a$.

The general sum connectivity index or first K_a index of a graph G is defined as $X^a(G) = \sum_{uv\in E(G)}[d_G(u)+d_G(v)]^a$. The above two topological indices were proposed in [1], [5] and [7].

In [2], Estrada et al. introduced the atom-bond connectivity index, which is defined as $ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$.

The geometric-arithmetic index of a graph G is defined as $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$. This index was proposed by Vukicevic and Furtula in [10].

The Kulli cycle windmill graph $C_{n+1}^{(m)}$ is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geqslant 3$ with a vertex K_1 in common. This graph is shown in Figure-1. The Kulli cycle windmill graph $C_4^{(m)}$ is a french windmill graph and it is denoted by $F_3^{(m)}$. For more details on french windmill graph, refer [4].



1 Kulli cycle windmill graph $C_{n+1}^{(m)}$.

2 Results

Theorem 2.1. *The Randic index of Kulli cycle windmill graph is*

$$\chi(C_{n+1}^{(m)}) = \left(\frac{1}{3} + \frac{1}{\sqrt{3mn}}\right) mn.$$

Proof. Let $G = C_{n+1}^{(m)}$, where $C_{n+1}^{(m)}$ is a Kulli cycle windmill graph. By algebraic method, we get $|V(G)| = mn + 1$ and $|E(G)| = 2mn$. We have two partitions of the vertex set $V(G)$ as follows:
$V_3 = \{v \in V(G) : d_G(v) = 3\}; |V_3| = mn$, and
$V_{mn} = \{v \in V(G) : d_G(v) = mn\}, |V_{2m}| = 1$.
Also we have two partitions of the edge set $E(G)$ as follows:
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}; |E_6| = mn$, and
$E_{mn+3} = \{uv \in E(G) : d_G(u) = mn, d_G(v) = 3\}; |E_{mn+3}| = mn$.

Now

$$\begin{aligned}
\chi(G) &= \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} \\
&= \sum_{uv \in E_6} \frac{1}{\sqrt{d_G(u)d_G(v)}} + \sum_{uv \in E_{mn+3}} \frac{1}{\sqrt{d_G(u)d_G(v)}} \\
&= mn\frac{1}{\sqrt{3 \times 3}} + mn\frac{1}{\sqrt{mn \times 3}} \\
&= \left(\frac{1}{3} + \frac{1}{\sqrt{3mn}}\right) mn.
\end{aligned}$$

□

Theorem 2.2. *The general Randic index of Kulli cycle windmill graph is* $\chi^a(C_{n+1}^{(m)}) = 3^a[3^a + (mn)^a]mn$.

Proof. Let $G = C_{n+1}^{(m)}$ be a Kulli cycle windmill graph. Now

$$\begin{aligned}
\chi^a(G) &= \sum_{uv \in E(G)} [d_G(u)d_G(v)]^a \\
&= \sum_{uv \in E_6} [d_G(u)d_G(v)]^a + \sum_{uv \in E_{mn+3}} [d_G(u)d_G(v)]^a \\
&= mn(3 \times 3)^a + mn(mn \times 3)^a \\
&= 3^a[3^a + (mn)^a]mn.
\end{aligned}$$

□

From Theorem 2.2, we have the following results.

Corollary 2.1. *The second Zagreb index of $C_{n+1}^{(m)}$ is*

$$M_2(C_{n+1}^{m}) = [3 + mn]3mn.$$

Corollary 2.2. *The second hyper Zagreb index of $C_{n+1}^{(m)}$ is*

$$HM_2(C_{n+1}^{m}) = [9 + (mn)^2]9mn.$$

Theorem 2.3. *The sum connectivity index of Kulli cycle windmill graph is*

$$X(C_{n+1}^{(m)}) = \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{mn+3}}\right)mn.$$

Proof. Let $G = C_{n+1}^{(m)}$ be a Kulli cycle windmill graph. Now

$$\begin{aligned} X(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}} \\ &= \sum_{uv\in E_6} \frac{1}{\sqrt{d_G(u)+d_G(v)}} + \sum_{uv\in E_{mn+3}} \frac{1}{\sqrt{d_G(u)+d_G(v)}} \\ &= mn\frac{1}{\sqrt{3+3}} + mn\frac{1}{\sqrt{mn+3}} \\ &= \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{mn+3}}\right)mn. \end{aligned}$$

□

Theorem 2.4. *The general sum connectivity index of Kulli cycle windmill graph is*

$$X^a(C_{n+1}^{(m)}) = [6^a + (mn+3)^a]mn.$$

Proof. Let $G = C_{n+1}^{(m)}$ be a Kulli cycle windmill graph. Now

$$\begin{aligned} X^a(G) &= \sum_{uv\in E(G)} [d_G(u)+d_G(v)]^a \\ &= \sum_{uv\in E_6} [d_G(u)+d_G(v)]^a + \sum_{uv\in E_{mn+3}} [d_G(u)+d_G(v)]^a \\ &= mn(3+3)^a + mn(mn+3)^a \\ &= [6^a + (mn+3)^a]mn. \end{aligned}$$

□

From the above Theorem, the following results are immediate

Corollary 2.3. *The first Zagreb index of $C_{n+1}^{(m)}$ is*

$$M_1(C_{n+1}^{m}) = [9+mn]mn.$$

Corollary 2.4. *The first hyper Zagreb index of $C_{n+1}^{(m)}$ is*

$$HM_1(C_{n+1}^{m}) = [(mn)^2 + 6mn + 45]mn.$$

Theorem 2.5. *The atom bond connectivity index of Kulli cycle windmill graph is*

$$ABC(C_{n+1}^{(m)}) = \left(\frac{2}{3} + \frac{\sqrt{mn+1}}{\sqrt{3mn}}\right)mn.$$

Proof. Let $G = C_{n+1}^{(m)}$, where $C_{n+1}^{(m)}$ is a Kulli cycle windmill graph. Now

$$\begin{aligned} ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\ &= \sum_{uv\in E_6} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} + \sum_{uv\in E_{mn+3}} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\ &= mn\sqrt{\frac{3+3-2}{3\times 3}} + mn\sqrt{\frac{mn+3-2}{mn\times 3}} \\ &= \left(\frac{2}{3} + \frac{\sqrt{mn+1}}{\sqrt{3mn}}\right) mn. \end{aligned}$$

□

Corollary 2.5. *The atom bond connectivity index of the french windmill graph $F_3^{(m)}$ is*

$$ABC(F_3^{(m)}) = ABC(C_4^{(m)}) = 2m + \sqrt{m(3m+1)}.$$

Theorem 2.6. *The geometric-arithmetic index of Kulli cycle windmill graph is*

$$GA(C_{n+1}^{(m)}) = \left(1 + \frac{2\sqrt{3mn}}{mn+3}\right) mn.$$

Proof. Let $G = C_{n+1}^{(m)}$, where $C_{n+1}^{(m)}$ is a Kulli cycle windmill graph.

Now

$$\begin{aligned} GA(G) &= \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} \\ &= \sum_{uv\in E_6} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} + \sum_{uv\in E_{mn+3}} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} \\ &= mn\frac{2\sqrt{3\times 3}}{3+3} + mn\frac{2\sqrt{mn\times 3}}{mn+3} \\ &= \left(1 + \frac{2\sqrt{3mn}}{mn+3}\right) mn. \end{aligned}$$

□

Corollary 2.6. *The geometric-arithmetic index of the french windmill graph $F_3^{(m)}$ is*

$$GA(F_3^{(m)}) = GA(C_4^{(m)}) = \left(1 + \frac{2\sqrt{m}}{m+1}\right) 3m.$$

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