Generalized solid capacitated transportation problem

Debiprasad Acharya

1 Introduction

Transportation problem is a special class of linear programming problem. It can be solved conventionally. But due to its special structure it can be solved in a different way. In 1941, Hitchcock [14] introduced classical transportation problem. In 1975, Klingman and Russel [15] developed a procedure for solving transportation problem with additional restriction. Several authors have studied such type of problem [2, 7, 8]. In 1994, Zheng et al. [16] considered transportation problem with upper limit constraints of the variables and with parameter. Several author have studied such type of problem [9, 11, 12, 18]. In 1964, generalized stepping stone method for machine loading problem was introduced by Elseman [17]. In 1975, Balachandran et al. [3] solved different types of generalized transportation problem. In 1987, Hadley [13] gave the detailed procedure for solving two dimensional generalized transportation problem. In 2000, Basu et al. [5] was introduced how to solve generalized solid transportation problem. Several author have studied such type of problem [1, 4, 6, 10].

But in practice there are many problems such as inventory problems, machine assignment problems, aircraft to routes etc. that have the common character as that of the proposed model. But this type of problem has been paid less attention. So, in this paper, generalized solid capacitated transportation problem with numerical example has discussed.
2 Problem Formulation

Let there be $m$ origins, $n$ destinations and $q$ products in a generalized solid capacitated transportation problem. Then the cost minimizing generalized solid capacitated transportation problem can be stated as:

$$
P: \quad \text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{q} r_{ijk} x_{ijk} \quad (2.1)$$

Subject to the constraints,

$$\sum_{k=1}^{q} d_{ijk} x_{ijk} \leq a_i; \quad 1 \leq i \leq m, \quad (2.2)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} = b_j; \quad 1 \leq j \leq n, \quad (2.3)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ijk} x_{ijk} \leq c_k; \quad 1 \leq k \leq q, \quad (2.4)$$

and $0 \leq x_{ijk} \leq R_{ijk}$ for $1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q,$

where, $x_{ijk}$ the amount of $k^{th}$ type of product transported from $i^{th}$ origin to $j^{th}$ destination,

$r_{ijk}$ the cost involved in transporting per unit of $k^{th}$ type of product transported from $i^{th}$ origin to $j^{th}$ destination,

$a_i$ the number of units available at the origin $-i$,

$b_j$ the number of units required at the destination $-j$,

$c_k$ requirement of the $k^{th}$ type of product,

$d_{ijk}^1$, $d_{ijk}^2$ are positive constants rather than unity and $R_{ijk} \geq 0$ be the capacitated restriction for the $k^{th}$ type of product on the route $i \rightarrow j$.

There are some special character in generalized solid transportation problem. They are stated below:

1. The rank of the co-efficient matrix of $[x_{ijk}]_{m \times n \times q}$ are in general $m + n + q$.
2. The integrability property of $x_{ijk}$ may not be hold.
3. The activity vector $p_{ijk} = d_{ijk}^1 e_i + e_{m+j} + d_{ijk}^2 e_{m+n+k} + \cdot$
4. The vectors in the loop are in general independent.

3 Solution Procedure

Introducing slack variables $s_{i,n+1,k}$ ($1 \leq i \leq m, 1 \leq k \leq q \geq 0$) and $s_{i,n+2,k}$ ($1 \leq i \leq m, 1 \leq k \leq q \geq 0$), the problem $P$ can be written as:

$$
P: \quad \text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n+2} \sum_{k=1}^{q} r_{ijk} x_{ijk} \quad (3.1)$$
Subject to the constraints,

\[
\sum_{j=1}^{n} \sum_{k=1}^{q} d_{ijk} x_{ijk} + \sum_{k=1}^{q} s_{i,n+1,k} = a_i; \quad 1 \leq i \leq m, \tag{3.2}
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = b_j; \quad 1 \leq j \leq n, \tag{3.3}
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ijk} x_{ijk} + \sum_{i=1}^{m} s_{i,n+2,k} = c_k; \quad 1 \leq k \leq q, \tag{3.4}
\]

and \( 0 \leq x_{ijk} \leq R_{ijk} \) for \( 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q \)

We first solve the problem \( P_1 \) from \( P \) by deleting the capacitated restriction.

\[
P_1: \quad \min Z = \sum_{i=1}^{m} \sum_{j=1}^{n+2} \sum_{k=1}^{q} r_{ijk} x_{ijk} \tag{3.5}
\]

Subject to the constraints,

\[
\sum_{j=1}^{n} \sum_{k=1}^{q} d_{ijk} x_{ijk} + \sum_{k=1}^{q} s_{i,n+1,k} = a_i; \quad 1 \leq i \leq m, \tag{3.6}
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = b_j; \quad 1 \leq j \leq n, \tag{3.7}
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ijk} x_{ijk} + \sum_{i=1}^{m} s_{i,n+2,k} = c_k; \quad 1 \leq k \leq q, \tag{3.8}
\]

and \( x_{ijk} \geq 0 \) for \( 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q \)

The initial solution can be obtained from any one of the standard method. Now the dual variables \( u_i \) (\( 1 \leq i \leq m \)), \( v_j \) (\( 1 \leq j \leq n \)) and \( w_k \) (\( 1 \leq k \leq q \)) are considered so that for the occupied cells we have,

\[
d_{ijk} u_i + v_j + d_{ijk} w_k = r_{ijk}; \quad 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q,
\]

\[
u_i = 0; \quad 1 \leq i \leq m, j = n + 1, 1 \leq k \leq q,
\]

\[w_k = 0; \quad 1 \leq i \leq m, j = n + 2, 1 \leq k \leq q,
\]

From hence, for the non-basic cells \( \Delta_{ijk} \) be defined as follows:

\[
\Delta_{ijk} = d_{ijk} u_i + v_j + d_{ijk} w_k - r_{ijk}; \quad 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq q,
\]

\[
= u_i; \quad 1 \leq i \leq m, j = n + 1, 1 \leq k \leq q,
\]

\[
= w_k; \quad 1 \leq i \leq m, j = n + 2, 1 \leq k \leq q,
\]

If \( \Delta_{ijk} \leq 0 \) for all non-occupied cells then the current solution is optimum. Otherwise we improve the solution as follows:

Let \( \Delta_{stp} = \max \{ \Delta_{ijk} : \Delta_{ijk} > 0 \} \) for \( 1 \leq i \leq m, 1 \leq j \leq n + 2, 1 \leq k \leq q \);

Then \( (s, t, p) \) cell enters into the basis. Calculation for leaving vector is as follows:
Let

\[ \sum_{(jk) \in B} d_{ijk} y_{stp} = d_{stp} e_s \]
\[ \sum_{(ik) \in B} y_{ijk} = e_{m+t} \]
\[ \sum_{(ij) \in B} d_{2ijk} y_{ijk} = d_{2stp} e_{m+n+p}; \]

where, \( p_{stp} = d_{stp} e_s + e_{m+t} + d_{2stp} e_{m+n+p} \), be the activity vector.

After determining, \( y_{ijk}^{*} \), we calculate \( \theta_{ijp}^{*} = \min \{ \frac{x_{ijk}^{*}}{y_{stp}} : y_{stp} \neq 0 \} \)

Let \( \theta = \frac{x_{ijk}^{*}}{y_{stp}^{*}} \). Then \((i_1, j_1, k_1)\) cell leaves the basis and the new solution is

\[ \hat{x}_{ijk} = x_{ijk} - y_{ijk} \times \theta \quad \text{for} \quad (i, j, k) \in B \quad \text{and} \quad (i, j, k) \neq (s, t, p) \]
\[ \theta = \frac{x_{ijk}^{*}}{y_{stp}^{*}} \quad \text{for} \quad (i, j, k) = (s, t, p) \]

Continuing this process until all \( \Delta_{ijk} \leq 0 \). Then optimum solution \( Z_1^0 \) is attained. Now we consider the capacitated restriction.

If \( 0 \leq x_{ijk} \leq R_{ijk} \) satisfies for all occupied cells then the current solution is the optimum solution of the problem \( P_1 \). Otherwise we proceed as follows:

Let it is not satisfied for \((s_1, t_1, p_1)\) cell in the basis, i.e. \( x_{s_1, t_1, p_1} > R_{s_1, t_1, p_1} \). Then substitute \( x_{s_1, t_1, p_1} = R_{s_1, t_1, p_1}, c_{s_1, t_1, p_1} = M \), where \( M \) is large positive number and

\[ a'_{s_1} = a_{s_1} - d_{s_1, t_1, p_1} R_{s_1, t_1, p_1} \]
\[ b'_{t_1} = b_{t_1} - R_{s_1, t_1, p_1} \]
\[ c'_{p_1} = c_{p_1} - d_{s_1, t_1, p_1} R_{s_1, t_1, p_1} \]

Then the problem \( P_1 \) can be modified as

\[
P_1^1 : \text{Min } Z = \sum_{i=1}^{m} \sum_{i \neq s_1}^{n} \sum_{j \neq t_1}^{n+2} \sum_{k \neq p_1}^{q} r_{ijk} x_{ijk} + M x_{s_1, t_1, p_1} \quad (3.9)
\]

Subject to the constraints,

\[
\sum_{j=1}^{n} \sum_{k=1}^{q} d_{1ijk} x_{ijk} \leq a_i; \quad 1 \leq i \leq m, i \neq s_1 \\
\sum_{j=1}^{n} \sum_{k=1}^{q} d_{1碇k} x_{s_1jk} \leq a'_{s_1}; \\
\sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} = b_j; \quad 1 \leq j \leq n, j \neq t_1 \\
\sum_{i=1}^{m} \sum_{k=1}^{q} x_{it_1k} = b'_{t_1}; \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{2ijk} x_{ijk} \leq c_k; \quad 1 \leq k \leq q, k \neq p_1 \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{2ijp_1} x_{ijp_1} \leq c'_{p_1}; \\
\]

This process is continued till \( 0 \leq x_{ijk} \leq R_{ijk} \) for \( 1 \leq i \leq m, 1 \leq j \leq n + 2, 1 \leq k \leq q \).
Let this process will be terminated at the $l$-th step. Let $S = \{x_{ijk}^B\}$ be the solution of $P_1^l$. Then the solution of $P$ is $S \cup_{ijk} R_{ijk}$. Where $\cup_{ijk}$ is the union taken over the all cells $(i, j, k)$ such that $x_{ijk} = R_{ijk}$ for $P_1^g (1 \leq g \leq l)$.

4 Algorithm

Step 1: Convert the problem $P_1$ from the problem $P$.

Step 2: Solve the problem $P_1$. If this solution satisfies all the conditions of the problem $P$, go to step 9, otherwise go to step 3.

Step 3: Let, $x_{s_1,t_1,p_1} > R_{s_1,t_1,p_1}$, then substitute

$$a_{s_1}' = a_{s_1} - d_{s_1,t_1,p_1} R_{s_1,t_1,p_1}$$

$$b_{t_1}' = b_{t_1} - R_{s_1,t_1,p_1}$$

$$c_{p_1}' = c_{p_1} - d_{s_1,t_1,p_1} R_{s_1,t_1,p_1}$$

Step 4: Set $b = 1$.

Step 5:

$$P_b^k : \text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{q} r_{ijk} x_{ijk} + M x_{s_1,t_1,p_1}$$

Where $M$ is large positive number.

Subject to the constraints,

$$\sum_{j=1}^{n} \sum_{k=1}^{q} d_{ijk} x_{ijk} \leq a_i; \quad 1 \leq i \leq m, i \neq s_1$$

$$\sum_{j=1}^{n} \sum_{k=1}^{q} d_{s_1,jk} x_{s_1,jk} \leq a_{s_1}'$$

$$\sum_{i=1}^{m} \sum_{k=1}^{q} x_{it_1,k} = b_{t_1}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij,t_1} \leq b_{t_1}'$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij,p_1} x_{ij,p_1} \leq c_{p_1}$$

If this solution satisfies all the conditions of the problem $P$, go to step 10, otherwise go to step 6.

Step 6: Set $b = b + 1$ and go to step 3. Continue this process until the solution of the problem $P$ has been reached.

Step 7: Set $b = l$, where $l$ is the number of iteration.

Step 8: Set the solution of $P_1^l = S$, where $S = \{x_{ijk}^B\}$ and go to step 10.

Step 9: The solution of $P$ is $S$ and go to Step 11.

Step 10: The solution of $P$ is $S \cup_{ijk} R_{ijk}$.

Step 11: Stop.
5 Numerical Example

We consider the following problem in Table 5, and we consider, 

The values of \((r_{ijk}, d_{1ijk}, d_{2ijk}, R_{ijk})\) are given in Table 5.

Optimum solution of the proposed problem is \(X^0 = \{x_{112} = 100, x_{122} = 300, x_{222} = 300, x_{211} = 100, x_{232} = 200\}\) and optimum cost \(Z^0 = 320\).

6 Conclusion

Nothing new will be discussed if the constraints

\[
\sum_{i=1}^{m} \sum_{k=1}^{q} x_{ijk} = b_j; \quad 1 \leq j \leq n,
\]

is changed into

\[
\sum_{i=1}^{m} \sum_{k=1}^{q} d_{i}^{3} x_{ijk} = b_j; \quad 1 \leq j \leq n,
\]

References