South Asian Journal of Mathematics 2014 , Vol. 4 (3) : 170 ∼ 180
www.sajm-online.com ISSN 2251-1512 RESEARCH ARTICLE

Some new properties of Fibonacci n-step

Manjusri Basu[①]*, Monojit Das[①]

① Department of Mathematics, University of Kalyani, Kalyani, W.B., India
E-mail: manjusri_basu@yahoo.com

Received: Dec-20-2013; Accepted: Jane-10-2014 *Corresponding author

Abstract We search the Fibonacci n-step numbers $F_k^{(n)}$ and tabulate these numbers for $k = 0, \pm1, \pm2, \ldots, \pm50$ and $n = 1, 2, 3, \ldots, 60$. Then we show some properties for generalizing these numbers for any integral values of k and for any $n \geqslant 1$.

Key Words Fibonacci numbers, Fibonacci n-step numbers

MSC 2010 11A51, 11B39

1 Introduction

The Fibonacci numbers F_k are defined by the second-order linear recurrence relation:

$$F_{k+1} = F_k + F_{k-1} \tag{1.1}$$

with the initial terms

$$F_0 = 0, \quad F_1 = 1.$$

This identity is called "Cassini formula" in honor of the well-known 17th century astronomer Giovanni Cassini (1625-1712) who derived this formula. Fibonacci n-step numbers $F_k^{(n)}$ are defined by linear recurrence relation of order $n > 1$:

$$F_k^{(n)} = F_{k-1}^{(n)} + F_{k-2}^{(n)} + \cdots + F_{k-n}^{(n)} \tag{1.2}$$

with n initial terms

$$F_0^{(n)} = F_1^{(n)} = \cdots = F_{n-2}^{(n)} = 0, \quad F_{n-1}^{(n)} = 1.$$

The numbers generated by the equation (1.2) are also known as the k-generalized Fibonacci numbers, which are discussed by Flores [4] in 1967. Equation (1.2) is equivalent to the recurrence relation of three terms

$$F_k^{(n)} = 2F_{k-1}^{(n)} - F_{k-n-1}^{(n)}.$$

In 1919, L. E. Dickson [3] discussed a long history of generalizations of the Fibonacci numbers. In 1960, E. P. Miles [6] used equation (1.2) and in 2003 Bengamin et. al. [1] briefly discuss a combinatorial interpretation of these n-step numbers.

When $n = 1$ the recurrence relation (1.1) gives the degenerate series $1, 1, 1, 1, 1, \ldots$.

Citation: Manjusri Basu, Monojit Das, Some new properties of Fibonacci n-step, South Asian J Math, 2014, 4(4), 170-180.

For $n = 2$, the recurrence relation (1.1) generates the usual Fibonacci numbers. Similarly for small values of n, e.g. $n = 3, 4, 5, 6, 7, 8$ etc. we obtain Tribonacci, Tetranacci (Quadranacci), Pentanacci (Pentacci), Hexanacci (Esanacci), Heptanacci, Octanacci etc. respectively.

For $k \geqslant 1$, $r_n = \lim_{k\to\infty} \frac{F_k^{(n)}}{F_{k-1}^{(n)}}$ exists, called n-anacci constant and is the real root $\geqslant 1$ of the equation

$$x^n - x^{n-1} - x^{n-2} - \cdots - x - 1 = 0,$$

or equivalently

$$x^n(2 - x) = 1.$$

For even n, there are exactly two real roots, one greater than 1 and one less than 1, and for odd n, there is exactly one real root, which is always $\geqslant 1$. Again I. Flores [4] proves that $\lim_{n\to\infty} r_n = 2$.

In 2005, Tony D. Noe et.al. [8] tabulate the values of k that yield the prime terms of the Fibonacci n-step sequences.

Definition 1.1. *A prime in the form* $2^n - 1$ *is called Mersenne prime.*

It is well known that every Mersenne prime appears in Fibonacci n-step sequences as $F_{2n}^{(n)}$. There are only 47 known Mersenne prime which are listed in Table 1 along with Fibonacci n-step number. $F_{86225218}^{(43112609)}$ is the largest known prime as well as Mersenne prime.

Table 1

Index	Mersenne prime	As Fibonacci $n-$ step number
1	$2^2 - 1$	$F_4^{(2)}$
2	$2^3 - 1$	$F_6^{(3)}$
3	$2^5 - 1$	$F_{10}^{(5)}$
4	$2^7 - 1$	$F_{14}^{(7)}$
5	$2^{13} - 1$	$F_{26}^{(13)}$
6	$2^{17} - 1$	$F_{34}^{(17)}$
7	$2^{19} - 1$	$F_{38}^{(19)}$
8	$2^{31} - 1$	$F_{62}^{(31)}$
9	$2^{61} - 1$	$F_{122}^{(61)}$
10	$2^{89} - 1$	$F_{178}^{(89)}$
11	$2^{107} - 1$	$F_{214}^{(107)}$
12	$2^{127} - 1$	$F_{254}^{(127)}$
13	$2^{521} - 1$	$F_{1042}^{(521)}$
14	$2^{607} - 1$	$F_{1214}^{(607)}$
15	$2^{1279} - 1$	$F_{2558}^{(1279)}$
16	$2^{2203} - 1$	$F_{4406}^{(2203)}$
17	$2^{2281} - 1$	$F_{4562}^{(2281)}$
18	$2^{3217} - 1$	$F_{6434}^{(3217)}$
19	$2^{4253} - 1$	$F_{8506}^{(4253)}$
20	$2^{4423} - 1$	$F_{8846}^{(4423)}$
21	$2^{9689} - 1$	$F_{19378}^{(9689)}$
22	$2^{9941} - 1$	$F_{19882}^{(9941)}$
23	$2^{11213} - 1$	$F_{22426}^{(11213)}$
24	$2^{19937} - 1$	$F_{39874}^{(19937)}$
25	$2^{21701} - 1$	$F_{43402}^{(21701)}$
26	$2^{23209} - 1$	$F_{46418}^{(23209)}$
27	$2^{44497} - 1$	$F_{88994}^{(44497)}$
28	$2^{86243} - 1$	$F_{172486}^{(86243)}$
29	$2^{110503} - 1$	$F_{221006}^{(110503)}$
30	$2^{132049} - 1$	$F_{264098}^{(132049)}$
31	$2^{216091} - 1$	$F_{432182}^{(216091)}$
32	$2^{756839} - 1$	$F_{1513678}^{(756839)}$
33	$2^{859433} - 1$	$F_{1718866}^{(859433)}$
34	$2^{1257787} - 1$	$F_{2515574}^{(1257787)}$
35	$2^{1398269} - 1$	$F_{2796538}^{(1398269)}$
36	$2^{2976221} - 1$	$F_{5952442}^{(2976221)}$
37	$2^{3021377} - 1$	$F_{6042754}^{(3021377)}$
38	$2^{6972593} - 1$	$F_{13945186}^{(6972593)}$
39	$2^{13466917} - 1$	$F_{26933834}^{(13466917)}$
40	$2^{20996011} - 1$	$F_{41992022}^{(20996011)}$
41	$2^{24036583} - 1$	$F_{48073166}^{(24036583)}$
42	$2^{25964951} - 1$	$F_{51929902}^{(25964951)}$
43	$2^{30402457} - 1$	$F_{60804914}^{(30402457)}$
44	$2^{32582657} - 1$	$F_{65165314}^{(32582657)}$
45	$2^{37156667} - 1$	$F_{74313334}^{(37156667)}$
46	$2^{42643801} - 1$	$F_{85287602}^{(42643801)}$
47	$2^{43112609} - 1$	$F_{86225218}^{(43112609)}$

The purpose of this paper is to tabulate the Fibonacci n-step numbers for $k = 0, \pm1, \pm2, \pm3, \ldots, \pm50$ and $n = 1, 2, 3, \ldots, 60$. Then shows some properties for generalizing these numbers for positive and negetive values of k and for $n \geqslant 1$. The following is Table 2-9:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
F_k^1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F_k^2	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181	6765	10946	17711
F_k^3	0	0	1	1	2	4	7	13	24	44	81	149	274	504	927	1705	3163	5768	10609	19513	35890	66012	121415
F_k^4	0	0	0	1	1	2	4	8	15	29	56	108	208	401	773	1490	2872	5536	10671	20569	39648	76424	147312
F_k^5	0	0	0	0	1	1	2	4	8	16	31	61	120	236	464	912	1793	3525	6930	13624	26784	52656	103519
F_k^6	0	0	0	0	0	1	1	2	4	8	16	32	63	125	248	492	976	1936	3840	7617	15109	29970	59448
F_k^7	0	0	0	0	0	0	1	1	2	4	8	16	32	64	127	253	504	1004	2000	3984	7936	15808	31489
F_k^8	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	255	509	1016	2048	4048	8080	16128
F_k^9	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256	511	1021	2040	4076	8144
F_k^{10}	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256	512	1023	2045	4088
F_k^{11}	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256	512	1024	2047
F_k^{12}	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256	512	1024
F_k^{13}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256	512
F_k^{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128	256
F_k^{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64	128
F_k^{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32	64
F_k^{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16	32
F_k^{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8	16
F_k^{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4	8
F_k^{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4
F_k^{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2
F_k^{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
F_k^{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F_k^{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{25}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{26}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{27}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{28}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{36}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k	23	24	25	26	27	28	29	30	31	32	33	34
F_k^1	1	1	1	1	1	1	1	1	1	1	1	1
F_k^2	28657	46368	75025	121393	196413	317811	514229	832040	1346269	2178309	3524578	5702887
F_k^3	223317	410744	7555476	1389537	2555757	4700770	8646064	15902591	29249425	53798080	98950096	181997601
F_k^4	283953	547337	1055026	2033628	3919944	7555935	14564533	28074040	54114452	104308960	201061985	387559437
F_k^5	203513	400096	786568	1546352	3040048	5976577	11749641	23099186	45411804	89277256	175514464	345052351
F_k^6	117920	233904	463968	920319	1825529	3621088	7182728	14247536	28261168	56058368	111196417	220567305
F_k^7	62725	124946	248888	495776	987568	1967200	3918592	7805695	15548665	30972384	61695880	122895984
F_k^8	32192	64256	128265	256005	510994	1019960	2035872	4063664	8111208	16190224	32316192	64504119
F_k^9	16272	32512	64960	129792	259328	518145	1035269	2068498	4132920	8257696	16499120	32965728
F_k^{10}	8172	16336	32656	65280	130496	260864	521472	1042432	2083841	4165637	8327186	16646200
F_k^{11}	4093	8184	16364	32720	65424	130816	261568	523008	1045760	2091008	4180992	
F_k^{12}	2048	4095	8189	16376	32748	65488	130960	261888	523712	1047296	2094336	4188160
F_k^{13}	1024	2048	4096	8191	16381	32760	65516	131024	262032	524032	1048000	2095872
F_k^{14}	512	1024	2048	4096	8192	16383	32765	65528	131052	262096	524176	1048320
F_k^{15}	256	512	1024	2048	4096	8192	16384	32767	65533	131064	262124	524240
F_k^{16}	128	256	512	1024	2048	4096	8192	16384	32768	65535	131069	262136
F_k^{17}	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131071
F_k^{18}	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536
F_k^{19}	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768
F_k^{20}	8	16	32	64	128	256	512	1024	2048	4096	8192	16384
F_k^{21}	4	8	16	32	64	128	256	512	1024	2048	4096	8192
F_k^{22}	2	4	8	16	32	64	128	256	512	1024	2048	4096
F_k^{23}	1	2	4	8	16	32	64	128	256	512	1024	2048
F_k^{24}	1	1	2	4	8	16	32	64	128	256	512	1024
F_k^{25}	0	1	1	2	4	8	16	32	64	128	256	512
F_k^{26}	0	0	1	1	2	4	8	16	32	64	128	256
F_k^{27}	0	0	0	1	1	2	4	8	16	32	64	128
F_k^{28}	0	0	0	0	1	1	2	4	8	16	32	64
F_k^{29}	0	0	0	0	0	1	1	2	4	8	16	32
F_k^{30}	0	0	0	0	0	0	1	1	2	4	8	16
F_k^{31}	0	0	0	0	0	0	0	1	1	2	4	8
F_k^{32}	0	0	0	0	0	0	0	0	1	1	2	4
$F^{33}{}_k$	0	0	0	0	0	0	0	0	0	1	1	2
F_k^{34}	0	0	0	0	0	0	0	0	0	0	1	1
F_k^{35}	0	0	0	0	0	0	0	0	0	0	0	1
F_k^{36}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{37}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{38}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{39}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{40}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{41}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{42}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{43}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{44}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{45}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{46}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{47}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{48}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{49}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{50}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0	0	0	0	0	0

k	35	36	37	38	39	40	41	42	43
F_k^1	1	1	1	1	1	1	1	1	1
F_k^2	9227465	14930352	24157817	39088169	63245986	102334155	165580141	267914296	433494437
F_k^3	334745777	615693474	1132436852	2082876103	3831006429	7046319384	12960201916	23837527729	43844049029
F_k^4	747044834	1439975216	2775641472	5350220959	10312882481	19878720128	38317465040	73859288608	142368356257
F_k^5	678355061	1333610936	2621810068	5154342880	10133171296	19921290241	39164225421	76994839906	151367869744
F_k^6	437513522	867844316	1721441096	3414621024	6773183680	13435170943	26649774581	52862035640	104856226964
F_k^7	244804400	487641600	971364608	1934923521	3854298377	7677624370	15293552860	30464209736	60683615072
F_k^8	128752233	256993472	512966984	1023898096	2043732528	4079353848	8142517472	16252718752	32440933385
F_k^9	65866496	131603200	262947072	525375999	1049716729	2097364960	4190597000	8372936304	16729373488
F_k^{10}	33276064	66519472	132973664	265816832	531372800	1062224128	2123405824	4244727807	8485289977
F_k^{11}	16715781	33423378	131603200	262947072	525375999	1049716729	2097364960	4190597000	8372936304
F_k^{12}	8375296	16748544	33492993	66977797	133939218	267845688	535625888	1071120816	2141979744
F_k^{13}	4191488	8382464	16763904	33525760	67047424	134086657	268156933	536281106	1072496696
F_k^{14}	2096576	4193024	8385792	16771072	33541120	67080192	134156288	268304384	536592385
F_k^{15}	1048464	2096896	4193728	8387328	16774400	33548288	67095552	134189056	268374016
F_k^{16}	524268	1048528	2097040	4194048	8388032	16776128	33551808	67103104	134205184
F_k^{17}	262141	524280	1048556	2097104	4194192	8388400	16776688	33553248	67106240
F_k^{18}	131072	262143	524285	1048568	2097132	4194268	8388508	16776984	33553904
F_k^{19}	65536	131072	262144	524287	1048573	2097144	4194284	8388560	16777104
F_k^{20}	32768	65536	131072	262144	524288	1048575	2097149	4194296	8388588
F_k^{21}	16384	32768	65536	131072	262144	524288	1048576	2097151	4194301
F_k^{22}	8192	16384	32768	65536	131072	262144	524288	1048576	2097152
F_k^{23}	4096	8192	16384	32768	65536	131072	262144	524288	1048576
F_k^{24}	2048	4096	8192	16384	32768	65536	131072	262144	524288
F_k^{25}	1024	2048	4096	8192	16384	32768	65536	131072	262144
F_k^{26}	512	1024	2048	4096	8192	16384	32768	65536	131072
F_k^{27}	256	512	1024	2048	4096	8192	16384	32768	65536
F_k^{28}	128	256	512	1024	2048	4096	8192	16384	32768
F_k^{29}	64	128	256	512	1024	2048	4096	8192	16384
F_k^{30}	32	64	128	256	512	1024	2048	4096	8192
F_k^{31}	16	32	64	128	256	512	1024	2048	4096
F_k^{32}	8	16	32	64	128	256	512	1024	2048
F_k^{33}	4	8	16	32	64	128	256	512	1024
F_k^{34}	2	4	8	16	32	64	128	256	512
F_k^{35}	1	2	4	8	16	32	64	128	256
F_k^{36}	1	1	2	4	8	16	32	64	128
F_k^{37}	0	1	1	2	4	8	16	32	64
F_k^{38}	0	0	1	1	2	4	8	16	32
F_k^{39}	0	0	0	1	1	2	4	8	16
F_k^{40}	0	0	0	0	1	1	2	4	8
F_k^{41}	0	0	0	0	0	1	1	2	4
F_k^{42}	0	0	0	0	0	0	1	1	2
F_k^{43}	0	0	0	0	0	0	0	1	1
F_k^{44}	0	0	0	0	0	0	0	0	1
F_k^{45}	0	0	0	0	0	0	0	0	0
F_k^{46}	0	0	0	0	0	0	0	0	0
F_k^{47}	0	0	0	0	0	0	0	0	0
F_k^{48}	0	0	0	0	0	0	0	0	0
F_k^{49}	0	0	0	0	0	0	0	0	0
F_k^{50}	0	0	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0	0	0

k	44	45	46	47	48	49	50
F_k^1	1	1	1	1	1	1	1
F_k^2	701408733	1134903170	1836311903	2971215073	4807526976	6643838879	11451365855
F_k^3	80641778674	148323355432	272809183135	501774317241	922906855808	1697490356184	3122171529233
F_k^4	274423830033	528968939938	1019620414836	1965381541064	3788394725871	7302365621709	14075762303480
F_k^5	297581396608	585029621920	1150137953599	2261111681777	4445228523648	8739089177552	17180596958496
F_k^6	207991012832	412567404640	818361625600	1623288080257	3219926385933	6386990736226	12669125245488
F_k^7	120879588544	240787812480	479640701439	955427104501	1903176584632	3791059616404	7551655023072
F_k^8	64753114537	129249235602	257985504220	514947110344	1027850488160	2051621622472	4095100727472
F_k^9	33425781248	66785696000	133439788800	266616630528	532707885057	1064366053385	2126634741810
F_k^{10}	16962252768	33907859336	67782442608	135498365744	270863757824	541461698816	1082392024832
F_k^{11}	8539642368	17075103744	34141847551	68266979321	136500535264	272934240136	545734852208
F_k^{12}	4283435776	8565824256	17129554176	34254920192	68501465088	136986181632	273938870271
F_k^{13}	2144862368	4289462704	8578401376	17155754752	34305222144	68610444288	137212506112
F_k^{14}	1073152005	2146238482	4292345912	8584429728	17167286960	34334573920	68667051264
F_k^{15}	536739840	1073463296	2146893825	4293722117	8587051046	17174102092	34347679944
F_k^{16}	268408320	536812544	1073616896	2147217408	4294336513	8588673026	1717695284700
F_k^{17}	134211968	268422912	536843776	1073683456	2147342336	4294684672	8589271040
F_k^{18}	67107680	134215104	268429696	536858368	1073710592	2147421184	4294817792
F_k^{19}	33554176	67108288	134216448	268432640	536863744	1073727488	2147448832
F_k^{20}	16777168	33554320	67108608	134217152	268433920	536867840	1073734144
F_k^{21}	8388600	16777196	33554384	67108752	134217408	268434816	536869248
F_k^{22}	4194303	8388605	16777208	33554412	67108800	134217600	268435104
F_k^{23}	2097152	4194304	8388607	16777213	33554424	67108844	134217680
F_k^{24}	1048576	2097152	4194304	8388608	16777215	33554429	67108856
F_k^{25}	524288	1048576	2097152	4194304	8388608	16777216	33554431
F_k^{26}	262144	524288	1048576	2097152	4194304	8388608	16777216
F_k^{27}	131072	262144	524288	1048576	2097152	4194304	8388608
F_k^{28}	65536	131072	262144	524288	1048576	2097152	4194304
F_k^{29}	32768	65536	131072	262144	524288	1048576	2097152
F_k^{30}	16384	32768	65536	131072	262144	524288	1048576
F_k^{31}	8192	16384	32768	65536	131072	262144	524288
F_k^{32}	4096	8192	16384	32768	65536	131072	262144
F_k^{33}	2048	4096	8192	16384	32768	65536	131072
F_k^{34}	1024	2048	4096	8192	16384	32768	65536
F_k^{35}	512	1024	2048	4096	8192	16384	32768
F_k^{36}	256	512	1024	2048	4096	8192	16384
F_k^{37}	128	256	512	1024	2048	4096	8192
F_k^{38}	64	128	256	512	1024	2048	4096
F_k^{39}	32	64	128	256	512	1024	2048
F_k^{40}	16	32	64	128	256	512	1024
F_k^{41}	8	16	32	64	128	256	512
F_k^{42}	4	8	16	32	64	128	256
F_k^{43}	2	4	8	16	32	64	128
F_k^{44}	1	2	4	8	16	32	64
F_k^{45}	1	1	2	4	8	16	32
F_k^{46}	0	1	1	2	4	8	16
F_k^{47}	0	0	1	1	2	4	8
F_k^{48}	0	0	0	1	1	2	4
F_k^{49}	0	0	0	0	1	1	2
F_k^{50}	0	0	0	0	0	1	1
F_k^{51}	0	0	0	0	0	1	1
F_k^{52}	0	0	0	0	1	1	2
F_k^{53}	0	0	0	0	0	0	1
F_k^{54}	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0

k	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13	−14	−15	−16	−17	−18	−19	−20
F_k^1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F_k^2	1	−1	2	−3	5	−8	13	−21	34	−55	89	−144	233	−377	610	−987	1597	−2584	4181	−6765
F_k^3	1	−1	0	2	−3	1	4	−8	5	7	−20	18	9	−47	56	0	−103	159	−56	−206
F_k^4	1	−1	0	0	2	−3	1	0	4	−8	5	−1	8	−20	18	−7	17	−48	56	−32
F_k^5	1	−1	0	0	0	2	−3	1	0	0	4	−8	5	−1	0	−12	0	18	−7	1
F_k^6	1	−1	0	0	0	0	2	−3	1	0	0	0	4	−8	5	−1	0	0	8	−20
F_k^7	1	−1	0	0	0	0	0	2	−3	1	0	0	0	0	4	−8	5	−1	0	0
F_k^8	1	−1	0	0	0	0	0	0	2	−3	1	0	0	0	0	0	4	−8	5	−1
F_k^9	1	−1	0	0	0	0	0	0	0	2	−3	1	0	0	0	0	0	0	4	−8
F_k^{10}	1	−1	0	0	0	0	0	0	0	0	2	−3	1	0	0	0	0	0	0	0
F_k^{11}	1	−1	0	0	0	0	0	0	0	0	0	2	−3	1	0	0	0	0	0	0
F_k^{12}	1	−1	0	0	0	0	0	0	0	0	0	0	2	−3	1	0	0	0	0	0
F_k^{13}	1	−1	0	0	0	0	0	0	0	0	0	0	0	2	−3	1	0	0	0	0
F_k^{14}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	2	−3	1	0	0	0
F_k^{15}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−3	1	0	0
F_k^{16}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−3	1	0
F_k^{17}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−3	1
F_k^{18}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	−3
F_k^{19}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
F_k^{20}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{21}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{22}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{23}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{24}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{25}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{26}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{27}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{28}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{29}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{30}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{31}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{32}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{33}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{34}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{35}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{36}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{37}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{38}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{39}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{40}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{41}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{42}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{43}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{44}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{45}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{46}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{47}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{48}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{49}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{50}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{51}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{52}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{53}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{54}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{55}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{56}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{57}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{58}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{59}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{60}	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

k	-21	-22	-23	-24	-25	-26	-27	-28	-29	-30	-31	-32	-33
F_k^1	1	1	1	1	1	1	1	1	1	1	1	1	1
F_k^2	10946	-17711	28657	-46368	75025	-121393	196413	-317811	514229	-832040	1346269	-2178309	3524578
F_k^3	421	-271	-356	1048	-963	-441	2452	-2974	81	5345	-8400	3136	10609
F_k^4	41	-113	160	-120	114	-267	433	-400	348	-648	433	-1233	-200
F_k^5	-24	12	36	-32	9	-49	48	60	-100	50	-47	85	72
F_k^6	18	-7	1	0	16	-48	56	-32	9	-1	32	-112	16
F_k^7	0	8	-20	18	-15	1	0	0	-4	-28	56	-32	9
F_k^8	0	0	0	0	8	-20	18	-7	1	0	0	0	16
F_k^9	5	-1	0	0	0	0	0	8	-20	18	-7	1	0
F_k^{10}	4	-8	5	-1	0	0	0	0	0	0	8	-20	18
F_k^{11}	0	0	4	-8	5	-1	0	0	0	0	0	0	0
F_k^{12}	0	0	0	0	4	-8	5	-1	0	0	0	0	0
F_k^{13}	0	0	0	0	0	0	4	-8	5	-1	0	0	0
F_k^{14}	0	0	0	0	0	0	0	0	4	-8	5	-1	0
F_k^{15}	0	0	0	0	0	0	0	0	0	0	4	-8	5
F_k^{16}	0	0	0	0	0	0	0	0	0	0	0	0	4
F_k^{17}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{18}	1	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{19}	-3	1	0	0	0	0	0	0	0	0	0	0	0
F_k^{20}	2	-3	1	0	0	0	0	0	0	0	0	0	0
F_k^{21}	0	2	-3	1	0	0	0	0	0	0	0	0	0
F_k^{22}	0	0	2	-3	1	0	0	0	0	0	0	0	0
F_k^{23}	0	0	0	2	-3	1	0	0	0	0	0	0	0
F_k^{24}	0	0	0	0	2	-3	1	0	0	0	0	0	0
F_k^{25}	0	0	0	0	0	2	-3	1	0	0	0	0	0
F_k^{26}	0	0	0	0	0	0	2	-3	1	0	0	0	0
F_k^{27}	0	0	0	0	0	0	0	2	-3	1	0	0	0
F_k^{28}	0	0	0	0	0	0	0	0	2	-3	1	0	0
F_k^{29}	0	0	0	0	0	0	0	0	0	2	-3	1	0
F_k^{30}	0	0	0	0	0	0	0	0	0	0	2	-3	1
F_k^{31}	0	0	0	0	0	0	0	0	0	0	0	2	-3
F_k^{32}	0	0	0	0	0	0	0	0	0	0	0	0	2
F_k^{33}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{34}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{35}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{36}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{37}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{38}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{39}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{40}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{41}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{42}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{43}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{44}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{45}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{46}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{47}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{48}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{49}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{50}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0	0	0	0	0	0	0

k	-34	-35	-36	-37	-38	-39	-40	-41	-42	-43
F_k^1	1	1	1	1	1	1	1	1	1	1
F_k^2	-5762887	9227465	-14930352	24157817	-39088169	63245986	-102334155	165580141	-267914296	433494437
F_k^3	-22145	14672	18082	-54899	51489	21492	-127880	157877	-8505	-277252
F_k^4	-348	2914	-3599	833	-496	2577	-6513	5265	-1825	5650
F_k^5	-260	200	-144	217	59	-592	560	-488	578	-99
F_k^6	-120	160	-121	65	-256	432	-400	440	-402	251
F_k^7	-1	0	-8	-52	14	-120	50	-11	1	-16
F_k^8	-48	56	-32	9	-1	0	0	32	-112	160
F_k^9	0	0	0	16	-48	56	-32	9	-1	0
F_k^{10}	-7	1	0	0	0	0	0	16	-48	56
F_k^{11}	8	-20	18	-7	1	0	0	0	0	0
F_k^{12}	0	0	0	8	-20	18	-7	1	0	0
F_k^{13}	0	0	0	0	0	0	8	-20	18	-7
F_k^{14}	0	0	0	0	0	0	0	0	8	-20
F_k^{15}	-1	0	0	0	0	0	0	0	0	0
F_k^{16}	-8	5	-1	0	0	0	0	0	0	0
F_k^{17}	0	4	-8	5	-1	0	0	0	0	0
F_k^{18}	0	0	0	4	-8	5	-1	0	0	0
F_k^{19}	0	0	0	0	0	4	-8	5	-1	0
F_k^{20}	0	0	0	0	0	0	0	4	-8	5
F_k^{21}	0	0	0	0	0	0	0	0	0	4
F_k^{22}	0	0	0	0	0	0	0	0	0	0
F_k^{23}	0	0	0	0	0	0	0	0	0	0
F_k^{24}	0	0	0	0	0	0	0	0	0	0
F_k^{25}	0	0	0	0	0	0	0	0	0	0
F_k^{26}	0	0	0	0	0	0	0	0	0	0
F_k^{27}	0	0	0	0	0	0	0	0	0	0
F_k^{28}	0	0	0	0	0	0	0	0	0	0
F_k^{29}	0	0	0	0	0	0	0	0	0	0
F_k^{30}	0	0	0	0	0	0	0	0	0	0
F_k^{31}	1	0	0	0	0	0	0	0	0	0
F_k^{32}	-3	1	0	0	0	0	0	0	0	0
F_k^{33}	2	-3	1	0	0	0	0	0	0	0
F_k^{34}	0	2	-3	1	0	0	0	0	0	0
F_k^{35}	0	0	2	-3	1	0	0	0	0	0
F_k^{36}	0	0	0	2	-3	1	0	0	0	0
F_k^{37}	0	0	0	0	2	-3	1	0	0	0
F_k^{38}	0	0	0	0	0	2	-3	1	0	0
F_k^{39}	0	0	0	0	0	0	2	-3	1	0
F_k^{40}	0	0	0	0	0	0	0	2	-3	1
F_k^{41}	0	0	0	0	0	0	0	0	2	-3
F_k^{42}	0	0	0	0	0	0	0	0	0	2
F_k^{43}	0	0	0	0	0	0	0	0	0	0
F_k^{44}	0	0	0	0	0	0	0	0	0	0
F_k^{45}	0	0	0	0	0	0	0	0	0	0
F_k^{46}	0	0	0	0	0	0	0	0	0	0
F_k^{47}	0	0	0	0	0	0	0	0	0	0
F_k^{48}	0	0	0	0	0	0	0	0	0	0
F_k^{49}	0	0	0	0	0	0	0	0	0	0
F_k^{50}	0	0	0	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0	0	0	0

k	-44	-45	-46	-47	-48	-49	-50
F_k^1	1	1	1	1	1	1	1
F_k^2	-701408733	1134903170	-1836311903	2971215073	-4807526976	6643838879	-11451365855
F_k^3	443634	-174887	545999	1164520	-793408	4805816457	-4803858529
F_k^4	-15673	17043	-8915	13125	-36856	49689	-34873
F_k^5	-1243	1912	-1636	1644	-776	-2387	5067
F_k^6	-577	1120	-1232	1280	-1244	904	-1405
F_k^7	-96	332	-380	220	-72	13	-33
F_k^8	-120	50	-11	1	0	64	-256
F_k^9	0	0	32	-112	160	-120	50
F_k^{10}	-32	9	-1	0	0	0	0
F_k^{11}	0	16	48	56	-32	9	-1
F_k^{12}	0	0	0	0	0	16	-48
F_k^{13}	1	0	0	0	0	0	0
F_k^{14}	-20	18	-7	1	0	0	0
F_k^{15}	0	0	8	-20	18	-7	1
F_k^{16}	0	0	0	0	0	8	-20
F_k^{17}	0	0	0	0	0	0	0
F_k^{18}	0	0	0	0	0	0	0
F_k^{19}	0	0	0	0	0	0	0
F_k^{20}	-1	0	0	0	0	0	0
F_k^{21}	-8	5	-1	0	0	0	0
F_k^{22}	0	4	-8	5	-1	0	0
F_k^{23}	0	0	0	4	-8	5	-1
F_k^{24}	0	0	0	0	0	4	-8
F_k^{25}	0	0	0	0	0	0	0
F_k^{26}	0	0	0	0	0	0	0
F_k^{27}	0	0	0	0	0	0	0
F_k^{28}	0	0	0	0	0	0	0
F_k^{29}	0	0	0	0	0	0	0
F_k^{30}	0	0	0	0	0	0	0
F_k^{31}	0	0	0	0	0	0	0
F_k^{32}	0	0	0	0	0	0	0
F_k^{33}	0	0	0	0	0	0	0
F_k^{34}	0	0	0	0	0	0	0
F_k^{35}	0	0	0	0	0	0	0
F_k^{36}	0	0	0	0	0	0	0
F_k^{37}	0	0	0	0	0	0	0
F_k^{38}	0	0	0	0	0	0	0
F_k^{39}	0	0	0	0	0	0	0
F_k^{40}	0	0	0	0	0	0	0
F_k^{41}	1	0	0	0	0	0	0
F_k^{42}	-3	1	0	0	0	0	0
F_k^{43}	2	-3	1	0	0	0	0
F_k^{44}	0	2	-3	1	0	0	0
F_k^{45}	0	0	2	-3	1	0	0
F_k^{46}	0	0	0	2	-3	1	0
F_k^{47}	0	0	0	0	2	-3	1
F_k^{48}	0	0	0	0	0	2	-3
F_k^{49}	0	0	0	0	0	0	2
F_k^{50}	0	0	0	0	0	0	0
F_k^{51}	0	0	0	0	0	0	0
F_k^{52}	0	0	0	0	0	0	0
F_k^{53}	0	0	0	0	0	0	0
F_k^{54}	0	0	0	0	0	0	0
F_k^{55}	0	0	0	0	0	0	0
F_k^{56}	0	0	0	0	0	0	0
F_k^{57}	0	0	0	0	0	0	0
F_k^{58}	0	0	0	0	0	0	0
F_k^{59}	0	0	0	0	0	0	0
F_k^{60}	0	0	0	0	0	0	0

2 Some properties of $F_k^{(n)}$

(1) For $n \geqslant 2$, $F_0^{(n)} = F_1^{(n)} = \cdots = F_{n-1}^{(n)} = 0, F_{n-1}^{(n)} = 1$ (initial condition),
$F_n^{(n)} = 2^0, F_{n+1}^{(n)} = 2^1, F_{n+2}^{(n)} = 2^2, F_{n+3}^{(n)} = 2^3, \ldots F_{2n-1}^{(n)} = 2^{n-1}, F_{2n}^{(n)} = 2^n - 1.$

(2) For $n \geqslant m(\geqslant 2)$, $F_{2n+m}^{(n)} = 2^{n+m} - a_m$,
where a_m satisfy the recurrence relation

$$a_m = 2a_{m-1} + 2^{m-1}, \quad a_0 = 1.$$

(3) For $n \geqslant 1$, $F_{-1}^{(n)} = 1$ and for $n \geqslant 2$, $F_{-2}^{(n)} = -1$.

(4) For $n \geqslant 2$, $F_{-n-1}^{(n)} = 2, F_{-n-2}^{(n)} = -3$ and $F_{-2n-2}^{(n)} = -8$.

(5) For $n \geqslant 3$, $F_{-k}^{(n)} = 0$ for $k = 3, 4, 5, \ldots$; $F_{-2n-1}^{(n)} = 4$ and $F_{-2n-3}^{(n)} = 5$.

(6) For $n \geqslant m(\geqslant 4)$, $F_{(-(m-3)n-m)}^{(n)} = 0$, $F_{(-(m-3)n-m-1)}^{(n)} = 0$, $F_{(-(m-3)n-m-2)}^{(n)} = 0$, $\ldots$ $F_{(-(m-2)n)}^{(n)} = 0$.

All these poperties can easily proved by induction using the equation (1.2) and Table 2 - Table 9.

References

1 A. T. Bengamin, J. J. Quinn, *The Fibonacci numbers-exposed more discretely*, Math. Mag. 76 (2003), 182-192.
2 M. Catalani, *Polymatrix and generalized polynacci numbers*, published electronically at www.arXiv.org/math/0210201
3 L. E. Dickson, *History of the theory of numbers*, AMS Chelsea vol. 1, 1919.
4 I. Flores, *Direct calulation of K-generalized Fibonacci numbers*, Fibonacci Quart., 5(1967),259-266.
5 T. Koshy, *Fibonacci and Lucas numbers with applications*, John Wiley and Sons, NY, 2001.
6 E, p. Miles, Jr., *Generalized Fibonacci numbers and associated matrices*, Amer, Math. Monthly 67 (1960), 745-752.
7 E. W. Weisstein, *Fibonacci n-step number*, published electronically at mathworld.wolfram.com/Fibonacci n-Step Number.html.
8 Tony D. Noe, Jonathan Vost Post, *Primes in Fibonacci n-step and Lucas n-step Sequences*, Journal of Integer Sequences, vol. 8(2005), artcle 05.4.4.