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Some fuzzy soft topological properties based on fuzzy semi open soft sets

A. Kandil[1], O.A.E. Tantawy[2], S.A. El-Sheikh[3]*, A.M. Abd El-latif[3]

[1] Mathematics Department, Faculty of Science, Helwan University, Helwan, Egypt.
[2] Mathematics Department, Faculty of Science, Zagazig University, Zagazig, Egypt.
[3] Mathematics Department, Faculty of Education, Ain Shams University, Cairo, Egypt.
E-mail: sobhyelsheikh@yahoo.com

Received: Jane-20-2014; Accepted: Aug-1-2014 *Corresponding author

Abstract In the present paper, we continue the study on fuzzy soft topological spaces and investigate the properties of fuzzy semi open (closed) soft sets, fuzzy semi soft interior (closure), fuzzy semi continuous (open) soft functions and fuzzy semi separation axioms which are important for further research on fuzzy soft topology. In particular we study the relationship between fuzzy semi soft interior fuzzy semi soft closure. Moreover, we study the properties of fuzzy soft semi regular spaces and fuzzy soft semi normal spaces. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

Key Words Soft set, Fuzzy soft set, Fuzzy soft topological space, Fuzzy semi soft interior
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1 Introduction

The concept of soft sets was first introduced by Molodtsov [20] in 1999 as a general mathematical tool for dealing with uncertain objects. In [20, 21], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [18], the properties and applications of soft set theory have been studied increasingly [5, 14, 21]. Xiao et al.[30] and Pei and Miao [24] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [3, 4, 6, 8, 16, 17, 18, 19, 21, 22, 33]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].

Recently, in 2011, Shabir and Naz [27] initiated the study of soft topological spaces. They defined soft topology as a collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties.

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Min in [29] investigate some properties of these soft separation axioms. Kandil et al. [13] introduce the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. Maji et. al. [16] initiated the study involving both fuzzy sets and soft sets. In [7] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then many scientists such as X. Yang et. al. [31], improved the concept of fuzziness of soft sets. In [1], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse image s of fuzzy soft sets. Chang [10] introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X. Tanay et.al. [28] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [26] gave the definition f fuzzy soft topology over the initial universe set.

In the present paper, we introduce some new concepts in fuzzy soft topological spaces such as fuzzy semi open soft sets, fuzzy semi closed soft sets, fuzzy semi soft interior, fuzzy semi soft closure and fuzzy semi separation axioms. In particular, we study the relationship between fuzzy semi soft interior and fuzzy semi soft closure. Also, we study the properties of fuzzy soft semi regular spaces and fuzzy soft semi normal spaces. Moreover, we show that if every fuzzy soft point f_e is fuzzy semi closed soft set in a fuzzy soft topological space $(X, \mathfrak{T}, E)$, then $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_1- (resp. T_2-) space. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1. *[32] A fuzzy set A in a non-empty set X is characterized by a membership function $\mu_A : X \longrightarrow [0,1] = I$ whose value$\mu_A(x)$ represents the "degree of membership" of x in A for $x \in X$.*
Let I^X denotes the family of all fuzzy sets on X. If $A, B \in I^X$, then some basic set operations for fuzzy sets are given by Zadeh [32], as follows:

(1) $A \leqslant B \;\Leftrightarrow\; \mu_A(x) \leqslant \mu_B(x)\; \forall\, x \in X.$

(2) $A = B \;\Leftrightarrow\; \mu_A(x) = \mu_B(x)\; \forall\, x \in X.$

(3) $C = A \vee B \;\Leftrightarrow\; \mu_C(x) = \mu_A(x) \vee \mu_B(x)\; \forall\, x \in X.$

(4) $D = A \wedge B \;\Leftrightarrow\; \mu_D(x) = \mu_A(x) \wedge \mu_B(x)\; \forall\, x \in X.$

(5) $M = A' \;\Leftrightarrow\; \mu_M(x) = 1 - \mu_A(x)\; \forall\, x \in X.$

Definition 2.2. *[16] Let $A \subseteq E$. A pair (f, A), denoted by f_A, is called a fuzzy soft set over X , where f is a mapping given by $f : A \to I^X$ defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \overline{0}$ if $e \notin A$ and $\mu^e_{f_A} \neq \overline{0}$ if $e \in A$ where $\overline{0}(e) = 0\; \forall\, x \in X$. The family of all these fuzzy soft sets over X denoted by $FSS(X)_A$.*

Proposition 2.1. *[4] Every fuzzy set may be considered a soft set.*

Definition 2.3. *[25] The complement of a fuzzy soft set (f, A), denoted by $(f, A)'$, is defined by $(f, A)' = (f', A)$, $f'_A : E \to I^X$ is a mapping given by $\mu^e_{f'_A} = \overline{1} - \mu^e_{f_A}$ $\forall\, e \in A$ where $\overline{1}(e) = 1$ $\forall\, x \in X$. Clearly $(f'_A)' = f_A$.*

Definition 2.4. *[18] A fuzzy soft set f_A over X is said to be a NULL fuzzy soft set, denoted by $\tilde{0}_A$, if for all $e \in A$, $f_A(e) = \overline{0}$.*

Definition 2.5. *[18] A fuzzy soft set f_A over X is said to be an absolute fuzzy soft set, denoted by $\tilde{1}_A$, if for all $e \in A$, $f_A(e) = \overline{1}$. Clearly we have $(\tilde{1}_A)' = \tilde{0}_A$ and $(\tilde{0}_A)' = \tilde{1}_A$.*

Definition 2.6. *[25] Let f_A, $g_B \in FSS(X)_E$. Then f_A is fuzzy soft subset of g_B, denoted by $f_A \sqsubseteq g_B$, if $A \subseteq B$ and $\mu^e_{f_A} \subseteq \mu^e_{g_B}$ $\forall\, e \in A$, i.e. $\mu^e_{f_A}(x) \leqslant \mu^e_{g_B}(x)$ $\forall\, x \in X$ and $\forall\, e \in A$.*

Definition 2.7. *[25]. The union of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cup B$ and for all $e \in C$,
$h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \vee \mu^e_{g_B}$ $\forall e \in C$. Here we write $h_C = f_A \sqcup g_B$.*

Definition 2.8. *[25]. The intersection of two fuzzy soft sets f_A and g_B over the common universe X is also a fuzzy soft set h_C, where $C = A \cap B$ and for all $e \in C$,
$h_C(e) = \mu^e_{h_c} = \mu^e_{f_A} \wedge \mu^e_{g_B}$ $\forall e \in C$. Here we write $h_C = f_A \sqcap g_B$.*

Theorem 2.1. *[3]. Let $\{(f, A)_j : j \in J\} \subseteq FSS(X)_E$. Then the following statements hold,*

(1) $[\sqcup_{j \in J}(f, A)_j]' = \sqcap_{j \in J}(f, A)'_j$,

(2) $[\sqcap_{j \in J}(f, A)_j]' = \sqcup_{j \in J}(f, A)'_j$.

Definition 2.9. *[25]. Let $\mathfrak{T}$ be a collection of fuzzy soft sets over a universe X with a fixed set of parameters E, then $\mathfrak{T}$ is called a fuzzy soft topology on X if*

(1) *$\tilde{1}_E, \tilde{0}_E \in \mathfrak{T}$, where $\tilde{0}_E(e) = \overline{0}$ and $\tilde{1}_E(e) = \overline{1}$, $\forall e \in E$,*

(2) *The union of any members of $\mathfrak{T}$, belongs to $\mathfrak{T}$,*

(3) *The intersection of any two members of $\mathfrak{T}$, belongs to $\mathfrak{T}$.*

The triplet $(X, \mathfrak{T}, E)$ is called a fuzzy soft topological space over X. Also, each member of $\mathfrak{T}$ is called a fuzzy open soft in $(X, \mathfrak{T}, E)$. We denote the set of all fuzzy open soft sets by $FOS(X, \mathfrak{T}, E)$, or $FOS(X)$.

Definition 2.10. *[25] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. A fuzzy soft set f_A over X is said to be fuzzy closed soft set in X, if its relative complement f'_A is fuzzy open soft set. We denote the set of all fuzzy closed soft sets by $FCS(X, \mathfrak{T}, E)$, or $FCS(X)$.*

Definition 2.11. *[23] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. The fuzzy soft closure of f_A, denoted by $Fcl(f_A)$ is the intersection of all fuzzy closed soft super sets of f_A. i.e.,
$Fcl(f_A) = \sqcap\{h_D : h_D \; is \; fuzzy \; closed \; soft \; set \; and \; f_A \sqsubseteq h_D\}$.
The fuzzy soft interior of g_B, denoted by $Fint(g_B)$ is the fuzzy soft union of all fuzzy open soft subsets of g_B.i.e.,
$Fint(g_B) = \sqcup\{h_D : h_D \; is \; fuzzy \; open \; soft \; set \; and \; h_D \sqsubseteq g_B\}$.*

Definition 2.12. *[15] The fuzzy soft set $f_A \in FSS(X)_E$ is called fuzzy soft point if there exist $x \in X$ and $e \in E$ such that $\mu^e_{f_A}(x) = \alpha$ $(0 < \alpha \leqslant 1)$ and $\mu^e_{f_A}(y) = \overline{0}$ for each $y \in X - \{x\}$, and this fuzzy soft point is denoted by x^e_α or f_e.*

Definition 2.13. *[15] The fuzzy soft point x^e_α is said to be belonging to the fuzzy soft set (g, A), denoted by $x^e_\alpha \tilde{\in} (g, A)$, if for the element $e \in A$, $\alpha \leqslant \mu^e_{g_A}(x)$.*

Theorem 2.2. *[15] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and f_e be a fuzzy soft point. Then the following properties hold:*

(1) *If $f_e \tilde{\in} g_A$, then $f_e \tilde{\notin} g'_A$;*

(2) $f_e \tilde{\in} g_A \nRightarrow f'_e \tilde{\in} g'_A$;

(3) *Every non-null fuzzy soft set f_A can be expressed as the union of all the fuzzy soft points belonging to f_A.*

Definition 2.14. *[15] A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called a fuzzy soft neighborhood of the fuzzy soft point x^e_α if there exists a fuzzy open soft set h_C such that $x^e_\alpha \tilde{\in} h_C \sqsubseteq g_B$. A fuzzy soft set g_B in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is called a fuzzy soft neighborhood of the soft set f_A if there exists a fuzzy open soft set h_C such that $f_A \sqsubseteq h_C \sqsubseteq g_B$. The fuzzy soft neighborhood system of the fuzzy soft point x^e_α, denoted by $N_{\mathfrak{T}}(x^e_\alpha)$, is the family of all its fuzzy soft neighborhoods.*

Definition 2.15. *[15] Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $Y \subseteq X$. Let h^Y_E be a fuzzy soft set over (Y, E) such that $h^Y_E : E \to I^Y$ such that $h^Y_E(e) = \mu^e_{h^Y_E}$,*

$$\mu^e_{h^Y_E}(x) = \begin{cases} 1, \ x \in Y, \\ 0, \ x \notin Y. \end{cases}$$

Let $\mathfrak{T}_Y = \{h^Y_E \sqcap g_B : g_B \in \mathfrak{T}\}$, then the fuzzy soft topology $\mathfrak{T}_Y$ on $(Y, E)$is called fuzzy soft subspace topology for (Y, E) and $(Y, \mathfrak{T}_Y, E)$ is called fuzzy soft subspace of $(X, \mathfrak{T}, E)$. If $h^Y_E \in \mathfrak{T}$ (resp. $h^Y_E \in \mathfrak{T}'$), then $(Y, \mathfrak{T}_Y, E)$ is called fuzzy open (resp. closed) soft subspace of $(X, \mathfrak{T}, E)$.

Definition 2.16. *[23] Let $FSS(X)_E$ and $FSS(Y)_K$ be families of fuzzy soft sets over X and Y, respectively. Let $u : X \to Y$ and $p : E \to K$ be mappings. Then the map f_{pu} is called a fuzzy soft mapping from X to Y and denoted by $f_{pu} : FSS(X)_E \to FSS(Y)_K$ such that,*

(1) *If $f_A \in FSS(X)_E$. Then the image of f_A under the fuzzy soft mapping f_{pu} is the fuzzy soft set over Y defined by $f_{pu}(f_A)$, where $\forall\ k \in p(E)$, $\forall\ y \in Y$,*

$$f_{pu}(f_A)(k)(y) = \begin{cases} \bigvee_{u(x)=y} \ [\vee_{p(e)=k}(f_A(e))](x) & if\ x \in u^{-1}(y), \\ 0 & otherwise. \end{cases}$$

(2) *If $g_B \in FSS(Y)_K$, then the pre-image of g_B under the fuzzy soft mapping f_{pu} is the fuzzy soft set over X defined by $f^{-1}_{pu}(g_B)$, where $\forall\ e \in p^{-1}(K)$, $\forall\ x \in X$,*

$$f^{-1}_{pu}(g_B)(e)(x) = \begin{cases} g_B(p(e))(u(x)) & for\ p(e) \in B, \\ 0 & otherwise. \end{cases}$$

The fuzzy soft mapping f_{pu} is called surjective (resp. injective) if p and u are surjective (resp. injective), also it is said to be constant if p and u are constant.

Definition 2.17. *[23] Let* $(X, \mathfrak{T}_1, E)$ *and* $(Y, \mathfrak{T}_2, K)$ *be two fuzzy soft topological spaces and* $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ *be a fuzzy soft mapping. Then* f_{pu} *is called*

(1) *Fuzzy continuous soft if* $f_{pu}^{-1}(g_B) \in \mathfrak{T}_1 \; \forall \; (g_B) \in \mathfrak{T}_2$.

(2) *Fuzzy open soft if* $f_{pu}(g_A) \in \mathfrak{T}_2 \forall \; (g_A) \in \mathfrak{T}_1$.

Theorem 2.3. *[1] Let* $FSS(X)_E$ *and* $FSS(Y)_K$ *be two families of fuzzy soft sets. For the fuzzy soft function* $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$*, the following statements hold,*

(a) $f_{pu}^{-1}((g, B)') = (f_{pu}^{-1}(g, B))' \forall \; (g, B) \in FSS(Y)_K$.

(b) $f_{pu}(f_{pu}^{-1}((g, B))) \sqsubseteq (g, B) \forall \; (g, B) \in FSS(Y)_K$*. If* f_{pu} *is surjective, then the equality holds.*

(c) $(f, A) \sqsubseteq f_{pu}^{-1}(f_{pu}((f, A))) \forall \; (f, A) \in FSS(X)_E$*. If* f_{pu} *is injective, then the equality holds.*

(d) $f_{pu}(\tilde{0}_E) = \tilde{0}_K$, $f_{pu}(\tilde{1}_E) \sqsubseteq \tilde{1}_K$*. If* f_{pu} *is surjective, then the equality holds.*

(e) $f_{pu}^{-1}(\tilde{1}_K) = \tilde{1}_E$ *and* $f_{pu}^{-1}(\tilde{0}_K) = \tilde{0}_E$.

(f) *If* $(f, A) \sqsubseteq (g, A)$*, then* $f_{pu}(f, A) \sqsubseteq f_{pu}(g, A)$.

(g) *If* $(f, B) \sqsubseteq (g, B)$*, then* $f_{pu}^{-1}(f, B) \sqsubseteq f_{pu}^{-1}(g, B) \; \forall \; (f, B), (g, B) \in FSS(Y)_K$.

(h) $f_{pu}^{-1}(\sqcup_{j \in J}(f, B)_j) = \sqcup_{j \in J} f_{pu}^{-1}(f, B)_j$ *and* $f_{pu}^{-1}(\sqcap_{j \in J}(f, B)_j) = \sqcap_{j \in J} f_{pu}^{-1}(f, B)_j, \forall \; (f, B)_j \in FSS(Y)_K$.

(I) $f_{pu}(\sqcup_{j \in J}(f, A)_j) = \sqcup_{j \in J} f_{pu}(f, A)_j$ *and* $f_{pu}(\sqcap_{j \in J}(f, A)_j) \sqsubseteq \sqcap_{j \in J} f_{pu}(f, A)_j \; \forall \; (f, A)_j \in FSS(X)_E$*. If* f_{pu} *is injective, then the equality holds.*

Definition 2.18. *[12] Let* (X, τ, E) *be a soft topological space and* $F_A \in SS(X)_E$*. If* $F_A \tilde{\subseteq} cl(int(F_A))$*, then* F_A *is called semi open soft set. We denote the set of all semi open soft sets by* $SOS(X, \tau, E)$*, or* $SOS(X)$ *and the set of all semi closed soft sets by* $SCS(X, \tau, E)$*, or* $SCS(X)$.

3 Fuzzy semi open (closed) soft sets

Various generalizations of closed and open soft sets in soft topological spaces were studied by Kandil et al. [12], but for fuzzy soft topological spaces such generalization have not been studied so far. In this section, we move one step forward to introduce fuzzy semi open and fuzzy semi closed soft sets and study various properties and notions related to these structures.

Definition 3.1. *Let* $(X, \mathfrak{T}, E)$ *be a fuzzy soft topological space and* $f_A \in FSS(X)_E$*. If* $f_A \sqsubseteq Fcl(Fint(f_A))$*, then* f_A *is called fuzzy semi open soft set. We denote the set of all fuzzy semi open soft sets by* $FSOS(X, \mathfrak{T}, E)$*, or* $FSOS(X)$ *and the set of all fuzzy semi closed soft sets by* $FSCS(X, \mathfrak{T}, E)$*, or* $FSCS(X)$.

Theorem 3.1. *Let* $(X, \mathfrak{T}, E)$ *be a fuzzy soft topological space and* $f_A \in FSOS(X)$*.Then*

(1) *Arbitrary fuzzy soft union of fuzzy semi open soft sets is fuzzy semi open soft.*

(2) *Arbitrary fuzzy soft intersection of fuzzy semi closed soft sets is fuzzy semi closed soft.*

Proof.

(1) Let $\{(f,A)_j : j \in J\} \subseteq FSOS(X)$. Then $\forall\ j \in J$, $(f,A)_j \sqsubseteq Fcl(Fint((f,A)_j))$. It follows that, $\sqcup_j (f,A)_j \sqsubseteq \sqcup_j (Fcl(Fint((f,A)_j))) = Fcl(\sqcup_j Fint(f,A)_j) \sqsubseteq Fcl(Fint(\sqcup_j (f,A)_j)$. Hence, $\sqcup_j (f,A)_j \in FSOS(X)\ \forall\ j \in J$.

(2) By a similar way.

Theorem 3.2. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then:*

(1) *$f_A \in FSOS(X)$ if and only if $Fcl(f_A) = Fcl(Fint(f_A))$.*

(2) *If $g_B \in \mathfrak{T}$, then $g_B \sqcap Fcl(f_A) \sqsubseteq Fcl(g_B \sqcap g_B)$.*

Proof. Immediate.

Theorem 3.3. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then:*

(1) *$f_A \in FSOS(X)$ if and only if there exists $g_B \in \mathfrak{T}$ such that $g_B \sqsubseteq f_A \sqsubseteq Fcl(g_B)$.*

(2) *If $f_A \in FSOS(X)$ and $f_A \sqsubseteq h_D \sqsubseteq Fcl(f_A)$, then $h_D \in FSOS(X)$.*

Proof.

(1) Let $f_A \in FSOS(X)$. Then $f_A \sqsubseteq Fcl(Fint(f_A))$. Take $g_B = Fint(f_A) \in \mathfrak{T}$. So, we have $g_B \sqsubseteq f_A \sqsubseteq Fcl(g_B)$. Sufficiency, let $g_B \sqsubseteq f_A \sqsubseteq Fcl(g_B)$ for some $g_B \in \mathfrak{T}$. Then $g_B \sqsubseteq Fint(f_A)$. It follows that, $Fcl(g_B) \sqsubseteq Fcl(Fint(f_A))$. Thus, $f_A \sqsubseteq Fcl(g_B) \sqsubseteq Fcl(Fint(f_A))$. Therefore, $f_A \in FSOS(X)$.

(2) Let $f_A \in FSOS(X)$. Then $g_B \sqsubseteq f_A \sqsubseteq Fcl(g_B)$ for some $g_B \in \mathfrak{T}$. It follows that, $g_B \sqsubseteq f_A \sqsubseteq h_D$. Thus, $g_B \sqsubseteq h_D \sqsubseteq Fcl(f_A) \sqsubseteq Fcl(g_B)$. Hence, $g_B \sqsubseteq h_D \sqsubseteq Fcl(g_B)$ for some $g_B \in \mathfrak{T}$. Therefore, $h_D \in FSOS(X)$ by (1).

Definition 3.2. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, $f_A \in FSS(X)_E$ and $f_e \tilde{\in} FSS(X)_E$. Then*

(1) *f_e is called fuzzy semi interior soft point of f_A if $\exists\ g_B \in FSOS(X)$ such that $f_e \tilde{\in} g_B \sqsubseteq f_A$. The set of all fuzzy semi interior soft points of f_A is called the fuzzy semi soft interior of f_A and is denoted by $FSint(f_A)$ consequently, $FSint(f_A) = \sqcup\{g_B : g_B \sqsubseteq f_A,\ g_B \in FSOS(X)\}$.*

(2) *f_e is called fuzzy semi cluster soft point of f_A if $f_A \sqcap h_C \neq \tilde{0}_E\ \forall\ h_D \in FSOS(X)$. The set of all fuzzy semi cluster soft points of f_A is called fuzzy semi soft closure of f_A and denoted by $FScl(f_A)$. Consequently, $FScl(f_A) = \sqcap\{h_D : h_D \in FSCS(X),\ f_A \sqsubseteq h_D\}$.*

Theorem 3.4. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. Then the following properties are satisfied for the fuzzy semi interior operator, denoted by $FSint$.*

(1) *$FSint(\tilde{1}_E) = \tilde{1}_E$ and $FSint(\tilde{0}_E)) = \tilde{0}_E$.*

(2) *$FSint(f_A) \sqsubseteq (f_A)$.*

(3) *$FSint(f_A)$ is the largest fuzzy semi open soft set contained in f_A.*

(4) *If $f_A \sqsubseteq g_B$, then $FSint(f_A) \sqsubseteq FSint(g_B)$.*

(5) *$FSint(FSint(f_A)) = FSint(f_A)$.*

(6) $FSint(f_A) \sqcup FSint(g_B) \sqsubseteq FSint[(f_A) \sqcup (g_B)]$.

(7) $FSint[(f_A) \sqcap (g_B)] \sqsubseteq FSint(f_A) \sqcap FSint(g_B)$.

Proof. Obvious.

Theorem 3.5. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. Then the following properties are satisfied for the fuzzy semi closure operator, denoted by $FScl$.*

(1) $FScl(\tilde{1}_E) = \tilde{1}_E$ *and* $FScl(\tilde{0}_E) = \tilde{0}_E$.

(2) $(f_A) \sqsubseteq FScl(f_A)$.

(3) $FScl(f_A)$ *is the smallest fuzzy semi closed soft set contains* f_A.

(4) *If* $f_A \sqsubseteq g_B$, *then* $FScl(f_A) \sqsubseteq FScl(g_B)$.

(5) $FScl(FScl(f_A)) = FScl(f_A)$.

(6) $FScl(f_A) \sqcup FScl(g_B) \sqsubseteq FScl[(f_A) \sqcup (g_B)]$.

(7) $FScl[(f_A) \sqcap (g_B)] \sqsubseteq FScl(f_A) \sqcap FScl(g_B)$.

Proof. Immediate.

Lemma 3.1. *Every fuzzy open (resp. closed) soft set in a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is fuzzy semi open (resp. closed) soft.*

Proof. Let $f_A \in FOS(X)$. Then $Fint(f_A) = f_A$. Since $f_A \sqsubseteq Fcl(f_A)$, then $f_A \sqsubseteq Fcl(Fint(f_A))$. Thus, $f_A \in FSOS(X)$.

Remark 3.1. *The converse of Lemma 3.1 is not true in general as shown in the following example.*

Example 3.1. *Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A, B, C, D \subseteq E$ where $A = \{e_1, e_2\}$, $B = \{e_2, e_3\}$, $C = \{e_1, e_3\}$ and $D = \{e_2\}$. Let $\mathfrak{T} = \{\tilde{1}_E, \tilde{0}_E, f_{1A}, f_{2B}, f_{3D}, f_{4E}, f_{5B}, f_{6D}\}$ where $f_{1A}, f_{2B}, f_{3D}, f_{4E}, f_{5B}, f_{6D}$ are fuzzy soft sets over X defined as follows:*
$\mu^{e_1}_{f_{1A}} = \{a_{0.5}, b_{0.75}, c_{0.4}\}$, $\mu^{e_2}_{f_{1A}} = \{a_{0.3}, b_{0.8}, c_{0.7}\}$,
$\mu^{e_2}_{f_{2B}} = \{a_{0.4}, b_{0.6}, c_{0.3}\}$, $\mu^{e_3}_{f_{2B}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{3D}} = \{a_{0.3}, b_{0.6}, c_{0.3}\}$,
$\mu^{e_1}_{f_{4E}} = \{a_{0.5}, b_{0.75}, c_{0.4}\}$, $\mu^{e_2}_{f_{4E}} = \{a_{0.4}, b_{0.8}, c_{0.7}\}$, $\mu^{e_3}_{f_{4E}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{5B}} = \{a_{0.4}, b_{0.8}, c_{0.7}\}$, $\mu^{e_3}_{f_{5B}} = \{a_{0.2}, b_{0.4}, c_{0.45}\}$,
$\mu^{e_2}_{f_{6D}} = \{a_{0.3}, b_{0.8}, c_{0.7}\}$.
Then $\mathfrak{T}$ defines a fuzzy soft topology on X. Then, the fuzzy soft set k_E where:
$\mu^{e_1}_{k_E} = \{a_{0.4}, b_{0.3}, c_{0.2}\}$, $\mu^{e_2}_{k_E} = \{a_{0.6}, b_{0.9}, c_{0.7}\}$, $\mu^{e_3}_{k_E} = \{a_{0.2}, b_{0.3}, c_{0.1}\}$.
is fuzzy semi open soft set of $(X, \mathfrak{T}, E)$, but it is not fuzzy open soft.

Theorem 3.6. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)$. Then:*

(1) $FSint(f'_A) = \tilde{1} - [FScl(f_A)]$.

(2) $FScl(f'_A) = \tilde{1} - [FSint(f_A)]$.

Proof.

(1) Since $FScl(f_A) = \sqcap\{h_D : h_D \in FSCS(X),\ f_A \sqsubseteq h_D\}$. Then $\tilde{1} - FScl(f_A) = \sqcup\{h'_D : h'_D \in FSOS(X),\ h'_D \sqsubseteq f'_A\} = FSint(f'_A)$.

(2) By a similar way.

Theorem 3.7. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A, g_B \in FSS(X)_E$. If either $f_A \in FSOS(X)$ or $g_B \in FSOS(X)$. Then, $Fint(Fcl(f_A \sqcap g_B)) = Fint(Fcl(f_A)) \sqcap Fint(Fcl(f_A))$.*

Proof. Let $f_A, g_B \in FSS(X)_E$. Then we generally have $Fint(Fcl(f_A \sqcap g_B)) \sqsubseteq Fint(Fcl(f_A)) \sqcap Fint(Fcl(f_A))$. Suppose that $f_A \in FSOS(X)$. Then $Fcl(f_A) = Fcl(Fint(f_A))$ from Theorem 3.2 (1). Therefore,

$$\begin{aligned} Fint(Fcl(f_A)) \sqcap Fint(Fcl(g_B)) &\sqsubseteq Fint[Fcl(f_A) \sqcap Fint(Fcl(g_B))] \\ &= Fint[Fcl(Fint(f_A)) \sqcap Fint(Fcl(g_B))] \\ &\sqsubseteq Fint(Fcl[Fint(f_A) \sqcap Fint(Fcl(g_B))]) \\ &\sqsubseteq Fint(Fcl(Fint[Fint(f_A) \sqcap Fcl(g_B)])) \\ &\sqsubseteq Fint(Fcl(Fint(Fcl[Fint(f_A) \sqcap (g_B)]))) \\ &\sqsubseteq Fint(Fcl(Fint(Fcl[(f_A) \sqcap (g_B)]))) \\ &\sqsubseteq Fint(Fcl[(f_A) \sqcap (g_B)]) \end{aligned}$$

from Theorem 3.2 (2). If $g_B \in FSOS(X)$, the proof is similar. This completes The proof.

Theorem 3.8. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, $f_A \in FOS(X)$ and $g_B \in FSOS(X)$. Then $f_A \sqcap g_B \in FSOS(X)$.*

Proof. Let $f_A \in FOS(X)$ and $g_B \in FSOS(X)$. Then $f_A \sqcap g_B \sqsubseteq Fint(f_A) \sqcap Fcl(Fint(g_B)) = Fcl(Fint(f_A) \sqcap Fint(g_B)) = Fcl(Fint((f_A) \sqcap (g_B)))$ from Theorem 3.2 (2). Hence, $f_A \sqcap g_B \in FSOS(X)$.

Theorem 3.9. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then $f_A \in FSCS(X)$ if and only if $Fint(Fcl(f_A)) \sqsubseteq f_A$.*

Proof. Let $f_A \in FSCS(X)$. Then f'_A is a fuzzy semi open soft set. This means that, $f'_A \sqsubseteq Fcl(Fint(\tilde{1}_E - f_A)) = \tilde{1}_E - (Fint(Fcl(f_A)))$. Therefore, $Fint(Fcl(f_A)) \sqsubseteq f_A$. Conversely, let $Fint(Fcl(f_A)) \sqsubseteq f_A$. Then $\tilde{1}_E - f_A \sqsubseteq Fcl(Fint(\tilde{1}_E - f_A))$. Hence, $\tilde{1}_E - f_A$ is a fuzzy semi open soft set. Therefore, f_A is fuzzy semi closed soft set.

Corollary 3.1. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and $f_A \in FSS(X)_E$. Then $f_A \in FSCS(X)$ if and only if $f_A = f_A \sqcup Fint(Fcl(f_A))$.*

4 Fuzzy semi continuous soft functions

In [1], Karal et al. defined the notion of a mapping on classes of fuzzy soft sets, which is fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse image s of fuzzy soft sets. Kandil et al. [13] introduced some types of soft function in soft topological spaces. Here, we introduce the notions of fuzzy semi soft function in fuzzy soft topological spaces and study its basic properties.

Definition 4.1. *Let* $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ *be fuzzy soft topological spaces and* $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$ *be a fuzzy soft function. Then, the function* f_{pu} *is called;*

(1) *Fuzzy semi continuous soft if* $f_{pu}^{-1}(g_B) \in FSOS(X) \forall\, g_B \in \mathfrak{T}_2$.

(2) *Fuzzy semi open soft if* $f_{pu}(g_A) \in FSOS(Y) \forall\, g_A \in \mathfrak{T}_1$.

(3) *Fuzzy semi closed soft if* $f_{pu}(f_A) \in FSCS(Y) \forall\, f_A \in \mathfrak{T}_1'$.

(4) *Fuzzy irresolute soft if* $f_{pu}^{-1}(g_B) \in FSOS(X) \forall\, g_B \in FSOS(Y)$.

(5) *Fuzzy irresolute open soft if* $f_{pu}(g_A) \in FSOS(Y) \forall\, g_A \in FSOS(X)$.

(6) *Fuzzy irresolute closed soft if* $f_{pu}(f_A) \in FSCS(Y) \forall\, f_A \in FSCS(Y)$.

Example 4.1. *Let* $X = Y = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ *and* $A \subseteq E$ *where* $A = \{e_1, e_2\}$. *Let* $f_{pu} : (X, \mathfrak{T}_1, E) \rightarrow (Y, \mathfrak{T}_2, K)$ *be the constant soft mapping where* $\mathfrak{T}_1$ *is the indiscrete fuzzy soft topology and* $\mathfrak{T}_2$ *is the discrete fuzzy soft topology such that* $u(x) = a \;\forall\, x \in X$ *and* $p(e) = e_1 \;\forall\, e \in E$. *Let* f_A *be fuzzy soft set over* Y *defined as follows:*
$\mu_{f_A}^{e_1} = \{a_{0.1}, b_{0.5}, c_{0.6}\}$, $\mu_{f_A}^{e_2} = \{a_{0.6}, b_{0.2}, c_{0.5}\}$.
Then $f_A \in \mathfrak{T}_2$. *Now, we find* $f_{pu}^{-1}(f_A)$ *as follows:*
$f_{pu}^{-1}(f_A)(e_1)(a) = f_A(p(e_1))(u(a)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_1)(b) = f_A(p(e_1))(u(b)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_1)(c) = f_A(p(e_1))(u(c)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_2)(a) = f_A(p(e_2))(u(a)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_2)(b) = f_A(p(e_2))(u(b)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_2)(c) = f_A(p(e_2))(u(c)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_3)(a) = f_A(p(e_3))(u(a)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_3)(b) = f_A(p(e_3))(u(b)) = f_A(e_1)(a) = 0.1$,
$f_{pu}^{-1}(f_A)(e_3)(c) = f_A(p(e_3))(u(c)) = f_A(e_1)(a) = 0.1$.
Hence, $f_{pu}^{-1}(f_A) \notin FSOS(X)$. *Therefore,* f_{pu} *is not fuzzy semi continuous soft function.*

Theorem 4.1. *Every fuzzy continuous soft function is fuzzy semi continuous soft.*

Proof. Immediate from Theorem 3.1.

Theorem 4.2. *Let* $(X, \mathfrak{T}_1, E)$, $(Y, \mathfrak{T}_2, K)$ *be fuzzy soft topological spaces and* f_{pu} *be a soft function such that* $f_{pu} : FSS(X)_E \rightarrow FSS(Y)_K$. *Then the following are equivalent:*

(1) f_{pu} *is a fuzzy semi continuous soft function.*

(2) $f_{pu}^{-1}(h_B) \in FSCS(X) \;\forall\, h_B \in FCS(Y)$.

(3) $f_{pu}(FScl(g_A) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)) \;\forall\, g_A \in FSS(X)_E$.

(4) $FScl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B)) \;\forall\, h_B \in FSS(Y)_K$.

(5) $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FSint(f_{pu}^{-1}(h_B)) \;\forall\, h_B \in FSS(Y)_K$.

Proof.

(1) $\Rightarrow$ (2) Let h_B be a fuzzy closed soft set over Y. Then $h'_B \in FOS(Y)$ and $f_{pu}^{-1}(h'_B) \in FSOS(X)$ from Definition 4.1. Since $f_{pu}^{-1}(h'_B) = (f_{pu}^{-1}(h_B))'$ from Theorem 2.3. Thus, $f_{pu}^{-1}(h_B) \in FSCS(X)$.

(2) $\Rightarrow$ (3) Let $g_A \in FSS(X)_E$. Since $g_A \sqsubseteq f_{pu}^{-1}(f_{pu}(g_A)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A))) \in FSCS(X)$ from (2) and Theorem 2.3. Then $g_A \sqsubseteq FScl(g_A) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))$. Hence, $f_{pu}(FScl(g_A)) \sqsubseteq f_{pu}(f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A)))$ from Theorem 2.3. Thus, $f_{pu}(FScl(g_A)) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(g_A))$.

(3) $\Rightarrow$ (4) Let $h_B \in FSS(Y)_K$ and $g_A = f_{pu}^{-1}(h_B)$. Then $f_{pu}(FScl f_{pu}^{-1}(h_B)) \sqsubseteq Fcl_{\mathfrak{T}_2}(f_{pu}(f_{pu}^{-1}$ $(h_B)))$ From (3). Hence, $FScl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(f_{pu}(FScl(f_{pu}^{-1}(h_B)))) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(f_{pu}(f_{pu}^{-1}(h_B))))$ $\sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))$ from Theorem 2.3. Thus, $FScl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))$.

(4) $\Rightarrow$ (2) Let h_B be a fuzzy closed soft set over Y. Then $FScl(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(Fcl_{\mathfrak{T}_2}(h_B))\ \forall\ h_B \in FSS(Y)_K$ from (4). But clearly $f_{pu}^{-1}(h_B) \sqsubseteq FScl(f_{pu}^{-1}(h_B))$. This means that, $f_{pu}^{-1}(h_B) = FScl(f_{pu}^{-1}(h_B))$, and consequently $f_{pu}^{-1}(h_B) \in FSCS(X)$.

(1) $\Rightarrow$ (5) Let $h_B \in FSS(Y)_K$. Then $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \in FSOS(X)$ from (1). Hence, $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) = FSint(f_{pu}^{-1} Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FSint(f_{pu}^{-1}(h_B))$. Thus, $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) \sqsubseteq FSint(f_{pu}^{-1}(h_B))$.

(5) $\Rightarrow$ (1) Let h_B be a fuzzy open soft set over Y. Then $Fint_{\mathfrak{T}_2}(h_B) = h_B$ and $f_{pu}^{-1}(Fint_{\mathfrak{T}_2}(h_B)) = f_{pu}^{-1}((h_B)) \sqsubseteq FSint(f_{pu}^{-1}(h_B))$ from (5). But we have $FSint(f_{pu}^{-1}(h_B)) \sqsubseteq f_{pu}^{-1}(h_B)$. This means that, $FSint(f_{pu}^{-1}(h_B)) = f_{pu}^{-1}(h_B) \in FSOS(X)$. Thus, f_{pu} is a fuzzy semi continuous soft function.

Theorem 4.3. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and f_{pu} be a soft function such that $f_{pu} : FSS(X)_E \to FSS(Y)_K$. Then the following are equivalent:*

(1) *f_{pu} is a fuzzy semi open soft function.*

(2) *$f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FSint(f_{pu}(g_A))\ \forall\ g_A \in FSS(X)_E$.*

Proof.

(1) $\Rightarrow$ (2) Let $g_A \in FSS(X)_E$. Since $Fint_{\mathfrak{T}_1}(g_A) \in \mathfrak{T}_1$. Then, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \in FSOS(Y)\ \forall\ g_A \in \mathfrak{T}_1$ by (1). It follow that, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) = FSint(f_{pu} Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FSint(f_{pu}(g_A))$. Therefore, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) \sqsubseteq FSint(f_{pu}(g_A))\ \forall\ g_A \in FSS(X)_E$.

(2) $\Rightarrow$ (1) Let $g_A \in \mathfrak{T}_1$. By hypothesis, $f_{pu}(Fint_{\mathfrak{T}_1}(g_A)) = f_{pu}(g_A) \sqsubseteq FSint(f_{pu}(g_A)) \in FSOS(Y)$, but $FSint(f_{pu}(g_A)) \sqsubseteq f_{pu}(g_A)$. So, $FSint(f_{pu}(g_A)) = f_{pu}(g_A) \in FSOS(Y)\ \forall\ g_A \in \mathfrak{T}_1$. Hence, f_{pu} is a fuzzy semi open soft function.

Theorem 4.4. *Let $f_{pu} : FSS(X)_E \to FSS(Y)_K$ be a fuzzy semi open soft function. If $k_D \in FSS(Y)_K$ and $l_C \in \mathfrak{T}'_1$ such that $f_{pu}^{-1}(k_D) \sqsubseteq l_C$, then there exists $h_B \in FSCS(Y)$ such that $k_D \sqsubseteq h_B$ and $f_{pu}^{-1}(h_B) \sqsubseteq l_C$.*

Proof. Let $k_D \in FSS(Y)_K$ and $l_C \in \mathfrak{T}'_1$ such that $f_{pu}^{-1}(k_D) \sqsubseteq l_C$. Then $f_{pu}(l'_C) \sqsubseteq k'_D$ from Theorem 2.3 where $l'_C \in \mathfrak{T}_1$. Since f_{pu} is fuzzy semi open soft function. Then $f_{pu}(l'_C) \in FSOS(Y)$. Take $h_B = [f_{pu}(l'_C)]'$. Hence, $h_B \in FSCS(Y)$ such that $k_D \sqsubseteq h_B$ and $f_{pu}^{-1}(h_B) = f_{pu}^{-1}([f_{pu}(l'_C)]') \sqsubseteq f_{pu}^{-1}(k'_D)' = f_{pu}^{-1}(k_D) \sqsubseteq l_C$. This completes the proof.

Theorem 4.5. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and f_{pu} be a soft function such that $f_{pu} : FSS(X)_E \to FSS(Y)_K$. Then, the following are equivalent:*

(1) f_{pu} *is a fuzzy semi closed soft function.*

(2) $FScl(f_{pu}(h_A)) \sqsubseteq f_{pu}(Fcl_{\mathfrak{T}_1}(h_A)) \ \forall \ h_A \in FSS(X)_E$.

Proof. It follows immediately from Theorem 4.3.

5 Fuzzy soft semi separation axioms

Soft separation axioms for soft topological spaces were studied by Shabir and Naz [27]. Kandil et al. [13] introduced and studied the notions of soft semi separation axioms in soft topological spaces. Here we introduce the notions of fuzzy semi separation axioms in fuzzy soft topological spaces and study some of its basic properties.

Definition 5.1. *Two fuzzy soft points* $f_e = x^e_\alpha$ *and* $g_e = y^e_\beta$ *are said to be distinct if and only if* $x \neq y$.

Definition 5.2. *A fuzzy soft topological space* $(X, \mathfrak{T}, E)$ *is said to be a fuzzy soft semi* T_o*-space if for every pair of distinct fuzzy soft points* f_e, g_e *there exists a fuzzy semi open soft set containing one of the points but not the other.*

Examples 5.1. **(1)** *Let* $X = \{a, b\}$, $E = \{e_1, e_2\}$ *and* $\mathfrak{T}$ *be the discrete fuzzy soft topology on* X*. Then* $(X, \mathfrak{T}, E)$ *is fuzzy soft semi* T_o*-space.*

(2) *Let* $X = \{a, b\}$, $E = \{e_1, e_2\}$ *and* $\mathfrak{T}$ *be the indiscrete fuzzy soft topology on* X*. Then* $\mathfrak{T}$ *is not fuzzy soft semi* T_o*-space.*

Theorem 5.1. *A soft subspace* $(Y, \mathfrak{T}_Y, E)$ *of a fuzzy soft semi* T_o*-space* $(X, \mathfrak{T}, E)$ *is fuzzy soft semi* T_o.

Proof. Let h_e, g_e be two distinct fuzzy soft points in (Y, E). Then these fuzzy soft points are also in (X, E). Hence, there exists a fuzzy semi open soft set f_A in $\mathfrak{T}$ containing one of the fuzzy soft points but not the other. Thus, $h^Y_E \sqcap f_A$ is a fuzzy semi open soft set in $(Y, \mathfrak{T}_Y, E)$ containing one of the fuzzy soft points but not the other from Definition 2.15. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi T_o.

Definition 5.3. *Two fuzzy soft points* $f_e = x^e_\alpha$ *and* $g_e = y^e_\beta$ *are said to be distinct if and only if* $x \neq y$.

Definition 5.4. *A fuzzy soft topological space* $(X, \mathfrak{T}, E)$ *is said to be a fuzzy soft semi* T_1*-space if for every pair of distinct fuzzy soft points* f_e, g_e *there exist fuzzy semi open soft sets* f_A *and* g_B *such that* $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$*; and* $f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$.

Example 5.1. *Let* $X = \{a, b\}$, $E = \{e_1, e_2\}$ *and* $\mathfrak{T}$ *be the discrete fuzzy soft topology on* X*. Then* $(X, \mathfrak{T}, E)$ *is fuzzy soft semi* T_1*-space.*

Theorem 5.2. *A fuzzy soft subspace* $(Y, \mathfrak{T}_Y, E)$ *of a fuzzy soft semi* T_1*-space* $(X, \mathfrak{T}, E)$ *is fuzzy soft semi* T_1.

Proof. It is similar to the proof of Theorem 5.1.

Theorem 5.3. *If every fuzzy soft point of a fuzzy soft topological space* $(X, \mathfrak{T}, E)$ *is fuzzy semi closed soft, then* $(X, \mathfrak{T}, E)$ *is fuzzy soft semi* T_1.

Proof. Suppose that f_e and g_e be two distinct fuzzy soft points of (X, E). By hypothesis, f_e and g_e are fuzzy semi closed soft sets. Hence, f'_e and g'_e are distinct fuzzy semi open soft sets where $f_e \tilde{\in} g'_e$, $g_e \tilde{\notin} g'_e$; and $f_e \tilde{\notin} f'_e$, $g_e \tilde{\in} f'_e$. Therefore, $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_1.

Definition 5.5. *A fuzzy soft topological space $(X, \mathfrak{T}, E)$ is said to be a fuzzy soft semi T_2-space if for every pair of distinct fuzzy soft points f_e, g_e there exist disjoint fuzzy semi open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$.*

Example 5.2. *Let $X = \{a, b\}$, $E = \{e_1, e_2\}$ and $\mathfrak{T}$ be the discrete fuzzy soft topology on X. Then $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_2-space.*

Proposition 5.1. *For a fuzzy soft topological space $(X, \mathfrak{T}, E)$ we have:*
fuzzy soft semi T_2-space $\Rightarrow$ fuzzy soft semi T_1-space $\Rightarrow$ fuzzy soft semi T_o-space.

Proof.

(1) Let $(X, \mathfrak{T}, E)$ be a fuzzy soft semi T_2-space and f_e, g_e be two distinct fuzzy soft points. Then there exist disjoint fuzzy semi open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$ and $g_e \tilde{\in} g_B$. Since $f_A \sqcap g_B = \tilde{0}_E$. Then $f_e \tilde{\notin} g_B$ and $g_e \tilde{\notin} f_A$. Therefore, there exist fuzzy semi open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$; and $f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_1-space.

(2) Let $(X, \mathfrak{T}, E)$ be a fuzzy soft semi T_1-space and f_e, g_e be two distinct fuzzy soft points. Then there exist fuzzy semi open soft sets f_A and g_B such that $f_e \tilde{\in} f_A$, $g_e \tilde{\notin} f_A$; and $f_e f_e \tilde{\notin} g_B$, $g_e \tilde{\in} g_B$. Then we have a fuzzy semi open soft set containing one of the fuzzy soft point but not the other. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_o-space.

Theorem 5.4. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space . If $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_2-space, then for every pair of distinct fuzzy soft points f_e, g_e there exists a fuzzy semi closed soft set k_A such that containing one of the fuzzy soft points $g_e \tilde{\in} k_A$, but not the other $f_e \tilde{\notin} k_A$ and $g_e \tilde{\notin} FScl(k_A)$.*

Proof. Let f_e, g_e be two distinct fuzzy soft points. By assumption, there exists disjoint fuzzy semi open soft sets b_A and h_B such that $f_e \tilde{\in} b_A$, $g_e \tilde{\in} h_B$. Hence, $g_e \tilde{\in} b'_A$ and $f_e \tilde{\notin} b'_A$ from Theorem 2.2. Thus, b'_A is a fuzzy semi closed soft set containing g_e but not f_e and $f_e \tilde{\notin} FScl(b'_A) = b'_A$.

Theorem 5.5. *A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of fuzzy soft semi T_2-space $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_2.*

Proof. Let j_e, k_e be two distinct fuzzy soft points in (Y, E). Then, these fuzzy soft points are also in (X, E). Hence, there exist disjoint fuzzy semi open soft sets f_A and g_B in $\mathfrak{T}$ such that $j_e \in f_A$ and $k_e \in g_B$. Thus, $h^Y_E \sqcap f_A$ and $h^Y_E \sqcap g_B$ are disjoint fuzzy semi open soft sets in $\mathfrak{T}_Y$ such that $j_e \in h^Y_E \sqcap f_A$ and $k_e \in h^Y_E \sqcap g_B$. So, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi T_2.

Theorem 5.6. *If every fuzzy soft point of a fuzzy soft topological space $(X, \mathfrak{T}, E)$ is fuzzy semi closed soft, then $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_2.*

Proof. It similar to the proof of Theorem 5.3.

Definition 5.6. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, h_C be a fuzzy semi closed soft set and g_e be a fuzzy soft point such that $g_e \tilde{\notin} h_C$. If there exist disjoint fuzzy semi open soft sets f_S and f_W such that $g_e \tilde{\in} f_S$ and $g_B \sqsubseteq f_W$. Then $(X, \mathfrak{T}, E)$ is called fuzzy soft semi regular space. A fuzzy soft semi regular T_1-space is called a fuzzy soft semi T_3-space.*

Proposition 5.2. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space, h_C be a fuzzy semi closed soft set and g_e be a fuzzy soft point such that $g_e \tilde{\notin} h_C$. If $(X, \mathfrak{T}, E)$ is fuzzy soft semi regular space, then there exists a fuzzy semi open soft set f_A such that $g_e \tilde{\in} f_A$ and $f_A \sqcap h_C = \tilde{0}_E$.*

Proof. Obvious from Definition 5.6.

Theorem 5.7. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft semi regular space and be a fuzzy semi open soft set g_B such that $f_e \tilde{\in} g_B$. Then, there exists a fuzzy semi open soft set f_S such that $f_e \tilde{\in} f_S$ and $FScl(f_S) \sqsubseteq g_B$.*

Proof. Let g_B be a fuzzy semi open soft set containing a fuzzy soft point f_e in a fuzzy soft semi regular space $(X, \mathfrak{T}, E)$. Then g'_B is a fuzzy semi closed soft such that $f_e \tilde{\notin} g'_B$ from Theorem 2.2. By hypothesis, there exist disjoint fuzzy semi open soft sets f_S and f_W such that $f_e \tilde{\in} f_S$ and $g'_B \sqsubseteq f_W$. It follows that, $f'_W \sqsubseteq g_B$ and $f_S \sqsubseteq f'_W$. Thus, $FScl(f_S) \sqsubseteq f'_W \sqsubseteq g_B$. So, we have a fuzzy semi open soft set f_S containing f_e such that $FScl(f_S) \sqsubseteq g_B$.

Theorem 5.8. *Every fuzzy soft semi T_3-space, in which every fuzzy soft point is fuzzy semi closed soft, is fuzzy soft semi T_2-space.*

Proof. Let f_e, g_e be two distinct fuzzy soft points of a fuzzy soft semi T_3-space $(X, \mathfrak{T}, E)$. By hypothesis, g_e is fuzzy semi closed soft set and $f_e \tilde{\notin} g_e$. From the fuzzy soft semi regularity, there exist disjoint fuzzy semi open soft sets k_A and h_B such that $f_e \tilde{\in} k_A$ and $g_e \sqsubseteq h_B$. Thus, $f_e \tilde{\in} k_A$ and $g_e \tilde{\in} h_B$. Therefore, $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_2-space.

Theorem 5.9. *A fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft semi T_3-space $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_3.*

Proof. By Theorem 5.2, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi T_1-space. Now, we want to prove that $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi regular space. Let k_E be a fuzzy semi closed soft set in (Y, E) and g_e be a fuzzy soft point in (Y, E) such that $g_e \tilde{\notin} k_E$. Then, $k_E = h_E^Y \sqcap g_B$ for some fuzzy semi closed soft set in (X, E). Hence, $g_e \tilde{\notin} h_E^Y \sqcap g_B$. But $g_e \tilde{\in} h_E^Y$, so $g_e \tilde{\notin} g_B$. Since $(X, \mathfrak{T}, E)$ is fuzzy soft semi T_3. Then, there exist disjoint fuzzy semi open soft sets f_S and f_W in $\mathfrak{T}$ such that $g_e \tilde{\in} f_S$ and $g_B \sqsubseteq f_W$. It follows that, $h_E^Y \sqcap f_S$ and $h_E^Y \sqcap f_W$ are disjoint fuzzy semi open soft sets in $\mathfrak{T}_Y$ such that $g_e \tilde{\in} h_E^Y \sqcap f_S$ and $k_E \sqsubseteq h_E^Y \sqcap f_W$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi T_3.

Definition 5.7. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space and h_C, g_B be disjoint fuzzy semi closed soft sets. If there exist disjoint fuzzy semi open soft sets f_S and f_W such that $h_C \sqsubseteq f_S$, $g_B \sqsubseteq f_W$. Then $(X, \mathfrak{T}, E)$ is called fuzzy soft semi normal space. A fuzzy soft semi normal T_1-space is called a fuzzy soft semi T_4-space.*

Theorem 5.10. *Let $(X, \mathfrak{T}, E)$ be a fuzzy soft topological space. Then the following are equivalent:*

(1) *$(X, \mathfrak{T}, E)$ is a fuzzy soft semi normal space.*

(2) *For every fuzzy semi closed soft set h_C and fuzzy semi open soft set g_B such that $h_C \sqsubseteq g_B$, there exists a fuzzy semi open soft set f_S such that $h_C \sqsubseteq f_S$, $FScl(f_S) \sqsubseteq g_B$.*

Proof.

(1) ⇒ (2) Let h_C be a semi closed soft set and g_B be a fuzzy semi open soft set such that $h_C \sqsubseteq g_B$. Then h_C, g'_B are disjoint fuzzy semi closed soft sets. It follows by (1), there exist disjoint fuzzy semi open soft sets f_S and f_W such that $h_C \sqsubseteq f_S$, $g'_B \sqsubseteq f_W$. Now, $f_S \sqsubseteq f'_W$, so $FScl(f_S) \sqsubseteq FScl f'_W = f'_W$, where g_B is fuzzy semi open soft set. Also, $f'_W \sqsubseteq g_B$. Hence, $FScl(f'_S) \sqsubseteq f'_W \sqsubseteq g_B$. Thus, $h_C \sqsubseteq f_S$, $FScl(f_S) \sqsubseteq g_B$.

(2) ⇒ (1) Let g_A and g_B be disjoint fuzzy semi closed soft sets. Then $g_A \sqsubseteq g'_B$. By hypothesis, there exists a fuzzy semi open soft set f_S such that $g_A \sqsubseteq f_S$, $FScl(f_S) \sqsubseteq g'_B$. So $g_B \sqsubseteq [FScl(f_S)]'$, $g_A \sqsubseteq f_S$ and $[FScl(f_S)]' \sqcap f_S = \tilde{0}_E$, where f_S and $[FScl(f_S)]'$ are fuzzy semi open soft sets. Thus, $(X, \mathfrak{T}, E)$ is fuzzy soft semi normal space.

Theorem 5.11. *A fuzzy semi closed fuzzy soft subspace $(Y, \mathfrak{T}_Y, E)$ of a fuzzy soft semi normal space $(X, \mathfrak{T}, E)$ is fuzzy soft semi normal.*

Proof. Let g_A and g_B be disjoint fuzzy semi closed soft sets in $\mathfrak{T}_Y$. Then, $g_A = h_E^Y \sqcap f_C$ and $g_B = h_E^Y \sqcap f_D$ for some fuzzy semi closed soft sets f_C, f_D in (X, E). Hence, f_C, f_D are disjoint fuzzy semi closed soft sets in $\mathfrak{T}$. Since $(X, \mathfrak{T}, E)$ is fuzzy soft semi normal. Then there exist disjoint fuzzy semi open soft sets f_S and f_W in $\mathfrak{T}$ such that $f_C \sqsubseteq f_S$, $f_D \sqsubseteq f_W$. It follows that, $h_E^Y \sqcap f_S$ and $h_E^Y \sqcap f_W$ are disjoint fuzzy semi open soft sets in $\mathfrak{T}_Y$ such that $g_A = h_E^Y \sqcap f_C \sqsubseteq h_E^Y \sqcap f_S$ and $g_B = h_E^Y \sqcap f_D \sqsubseteq h_E^Y \sqcap f_W$. Therefore, $(Y, \mathfrak{T}_Y, E)$ is fuzzy soft semi normal.

Theorem 5.12. *Let $(X, \mathfrak{T}_1, E)$ and $(Y, \mathfrak{T}_2, K)$ be fuzzy soft topological spaces and $f_{pu} : SS(X)_E \to SS(Y)_K$ be a fuzzy soft function which is bijective, fuzzy irresolute soft and fuzzy irresolute open soft. If $(X, \mathfrak{T}_1, E)$ is a fuzzy soft semi normal space, then $(Y, \mathfrak{T}_2, K)$ is also a fuzzy soft semi normal space.*

Proof. Let f_A, g_B be disjoint fuzzy semi closed soft sets in Y. Since f_{pu} is fuzzy irresolute soft, then $f_{pu}^{-1}(f_A)$ and $f_{pu}^{-1}(g_B)$ are fuzzy semi closed soft set in X such that $f_{pu}^{-1}(f_A) \sqcap f_{pu}^{-1}(g_B) = f_{pu}^{-1}[f_A \sqcap g_B] = f_{pu}^{-1}[\tilde{0}_K] = \tilde{0}_E$ from Theorem 2.3. By hypothesis, there exist disjoint fuzzy semi open soft sets k_C and h_D in X such that $f_{pu}^{-1}(f_A) \sqsubseteq k_C$ and $f_{pu}^{-1}(g_B) \sqsubseteq h_D$. It follows that, $f_A = f_{pu}[f_{pu}^{-1}(f_A)] \sqsubseteq f_{pu}(k_C)$,$g_B = f_{pu}[f_{pu}^{-1}(g_B)] \sqsubseteq f_{pu}(h_D)$ from Theorem 2.3 and $f_{pu}(k_C) \sqcap f_{pu}(h_D) = f_{pu}[k_C \sqcap h_D] = f_{pu}[\tilde{0}_E] = \tilde{0}_K$ from Theorem 2.3. Since f_{pu} is fuzzy irresolute open soft function. Then $f_{pu}(k_C), f_{pu}(h_D)$ are fuzzy semi open soft sets in Y. Thus, $(Y, \mathfrak{T}_2, K)$ is a fuzzy soft semi normal space.

6 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov [20] and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of fuzzy soft topological spaces. We introduce the some new concepts in fuzzy soft topological spaces such as fuzzy semi open soft sets, fuzzy semi closed soft sets, fuzzy semi soft interior, fuzzy semi soft closure and fuzzy semi separation axioms and have established several interesting properties. Since the authors introduced topological structures on fuzzy soft sets [7, 11, 28], so the semi topological properties, which introduced by Kandil et al.[13], is generalized here to the fuzzy soft sets which will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [30, 24], we can use the results deducted from the studies on fuzzy soft topological space to improve these kinds of connections. We hope that the findings in this paper will help researcher enhance and promote the further study on fuzzy soft topology to carry out a general framework for their applications in practical life.

References

1 B. Ahmad and A. Kharal, Mappings on fuzzy soft classes, Adv. Fuzzy Syst. 2009, Art. ID 407890, 6 pp.

2 B. Ahmad and A. Kharal, Mappings on soft classes, New Math. Nat. Comput. 7 (3) (2011) 471-481.

3 B. Ahmad and A. Kharal, On fuzzy soft sets, Adv. Fuzzy Syst. 2009, Art. ID 586507, 6 pp.

4 H. Aktas and N. Cagman, Soft sets and soft groups, Information Sciences 1 (77)(2007) 2726-2735.

5 M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications 57 (2009) 1547-1553.

6 S. Atmaca and I.Zorlutuna, On fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics 5 (2013) 377-386.

7 Bakir Tanay and M. Burcl Kandemir, Topological structure of fuzzy soft sets, Computer and mathematics with applications (61)(2011), 2952-2957.

8 N. Ça gman, F. Çi tak and S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems 1(1)(2010) 21-35.

9 N. Ça gman and S. Enginoglu, Soft set theory and uni-Fint decision making, European Journal of Operational Research 207 (2010) 848-855.

10 C. L. Change, Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.

11 Jianyu Xiao, Minming Tong, Qi Fan and Su Xiao, Generalization of Belief and Plausibility Functions to Fuzzy Sets, Applied Mathematics Information Sciences 6 (2012), 697-703.

12 A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, Annals of Fuzzy Mathematics and Informatics 7 (2014) 181-196.

13 A.Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, Soft semi separation axioms and irresolute soft functions, Ann. Fuzzy Math. Inform. 8 (2) (2014) 305-318.

14 D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, Soft sets theory-based optimization, Journal of Computer and Systems Sciences Finternational 46 (6) (2007) 872-880.

15 J. Mahanta and P.K. Das, Results on fuzzy soft topological spaces, arXiv:1203.0634v1,2012.

16 P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 589-602.

17 P. K. Maji, R. Biswas and A. R. Roy, intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001) 677-691.

18 P. K. Maji,R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555-562.

19 P. Majumdar and S. K. Samanta, Generalised fuzzy soft sets, Computers and Mathematics with Applications 59 (2010) 1425-1432.

20 D. A. Molodtsov, Soft set theory-firs tresults, Computers and Mathematics with Applications 37 (1999) 19-31.

21 D.Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (1) (2006) 8-39.

22 A. Mukherjee and S. B. Chakraborty, On Fintuitionistic fuzzy soft relations, Bulletin of Kerala Mathematics Association 5 (1) (2008) 35-42.

23 B. Pazar Varol and H. Aygun, Fuzzy soft topology, Hacettepe Journal of Mathematics and Statistics 41 (3) (2012) 407-419.

24 D. Pei and D. Miao, From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol.2, 2005, pp. 617-621.

25 S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and informatics 3 (2) (2012) 305-311.

26 S. Roy and T. K. Samanta, An introduction to open and closed sets on fuzzy topological spaces, Annals of Fuzzy Mathematics and Informatics, 2012.

27 M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786-1799.

28 B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, Computer and Math. with appl. 61 (2011) 412-418.

29 Won Keun Min, A note on soft topological spaces, Computers and Mathematics with Applications 62 (2011) 3524-3528.

30 Z. Xiao, L. Chen, B. Zhong and S. Ye, Recognition for soft information based on the theory of soft sets, in: J. Chen (Ed.), Proceedings of ICSSSM-05, vol. 2, IEEE, 2005, pp. 1104-1106.

31 Yong Chan Kim and Jung Mi Ko, Fuzzy G-closure operators, commun Korean Math. Soc. 18(2)(2008), 325-340.

32 L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.

33 Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems 21 (2008) 941-945.