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Generalized Fixed point theorem in fuzzy metric spaces

Teki Sivakrishna①, D. Venugopalam①, M. Vijaya Kumar①*

① Department of Mathematics, Sri Vasavi Institute of Engineering and Technology (SVIET), Nandamuru, India
E-mail: greengirl_ias2001@yahoo.com

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Abstract The aim of this paper is to consider fixed point theorem for fuzzy metric spaces for weakly commuting mappings.

Key Words fuzzy 3-metric spaces, common fixed point

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1 Introduction

first introduced by L. A. Zadeh [20] The concept of a fuzzy set was and developed a basic frame work to treat mathematically the fuzzy phenomena or systems which due to in transact ndefiniteness, cannot themselves be characterized precisely. fuzzy metric spaces have been introduced byKramosil and Michalek [7] and George and Veersamani [3] modified the notion of fuzzy metric with help of continuous t-norms. Recently many have proved fixed point theorems involving fuzzy sets [1, 2, 4-6, 8-10,14, 16-19]. Vasuki [19] investigated same fixed point theorems in fuzzy metric spaces for R-weakly commuting mappings and part [12] introduced the notion of reciprocal continuity of mappings in metric spaces. Balasubramaniam etal and S. Muralishankar, R.P. Pant [1] proved the open problem of Rhoades [15] on the existence of a contractive definition which generals a fixed point but does not force the mapping to be continuous at the fixed point possesses an affirmative answer. We know that that 2-metric space is a real valued function of a point triples on a set X,which abstract properties were suggested by the area function in Euclidean spaces. Now it is natural to expect 3-Metric space, which is suggested by the volume function.

In the present paper we are proving a common fixed point theorem for fuzzy3-metrics paces for weakly commuting mappings

2 Preliminaries

Before starting the main result we need some basic definitions and basic results

Definition 2.1. A fuzzy set A in x is a function with domain X and values in [0,1].

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Definition 2.2. A binary operation $* : [0.1] \to [0.1]$ is called a continuous t-norm of $([0.1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leqslant C * d$ whenever $a \leqslant c$ and $b \leqslant d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.3. A Triplet $(x, M, *)$ is called a fuzzy metric space if x is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $x^2 \times [0, \infty)$ satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$

I. $M(x, y, t) > 0$

II. $M(x, y, t) = 1 \Rightarrow x = y$

III. $M(x, y, t) = M(y, x, t)$

IV. $M(x, y, t) * M(y, z, s) \leqslant M(x, z, t + s)$

V. $M(x, y.t) : [0, \infty) \to [0, 1]$ is continuous

VI. $\lim_{t\to\infty} M(x, y, t) = 1 \Rightarrow x, y \in X$.

Then M is called a fuzzy metric on X. A function $M(x, y, t)$ denote the degree of nearness between x and y with respect to t.

Example 2.4 (Induced Fuzzy metric) [3]. Every Metric space indices a fuzzy metric space. Let (x, d) be a metric space. Define $a * b = ab$ and $M(x, y, t) = (kt^n)/(kt^n + md(x, y))$.

The fuzzy metric induced by a metric d is referred to as a standard fuzzy metric

Definition 2.5. A binary operation $* : [0, 1]^4 \to [0, 1]$ is called a continuous t-norm if $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a_1 * b_1 * c_1 * d_1 \geqslant a_2 * b_2 * c_2 * d_2$ Whenever $a_1 \geqslant a_2, b_1 \geqslant b_2$, $c_1 \geqslant c_2$ and $d_1 \geqslant d_2$ for all $a_1, a_2, b_1, b_2, c_1, c_2$ and d_1, d_2 are in [0,1].

Definition 2.6. The 3-tuple $(X, M, *)$ is called a fuzzy 3-metric space if X is an arbitrary set, $*$ is continuous t-norm monodies and M is a fuzzy set in $X^4 x[0, \infty]$ satisfying the following conditions:

I. $M(x, y, z, wt) = 0$

II. $M(x, y, z, w, t) = 1$ for all $t > 0$

III. $M(x, y, z, w, t) = M(x, w, z, y, t)$, $M(z, w, x, y, t)$

IV. $M(x, y, z, wt_1 + t_2 + t_3 + t_4) \geqslant M(x, y, z, u, t_1)M(x, y, u, w, t_1)M(x, u, z, w, t_1)M(u, y, z, w, t_1)$

V. $M(x, y, z, wt) =: [0, 1) \to [0, 1]$ is continuous for all $x, y, z, u, w, \in X, t_1, t_2, t_3, t_4 > 0$

Definition 2.7. Let $(X, M, *)$ be a fuzzy 3-metric space. A sequence $\{X_n\}$ in fuzzy 3-metric space X

1) is said to be convergent to a point $x \in X$, if $\lim_{n\to\infty} M(x_n, x, a, b, t) = 1$

(2) is called a Cauchy sequence, if $\lim_{n\to\infty} M(x_{n+p}, x_n, a, b, t) = 1$ for all $a, b \in X$ and $t, p > 0$

(3) every Cauchy sequence is convergent is said to be complete.

Definition 2.8. A function M is continuous in fuzzy 3-metric space if $x_n \to x$ and $y_n \to y$, $\lim_{t\to\infty}$ $M(x_n, y_n, a, b, t) = M(x, y, a, t)$ for a, b, belong to X and $t > 0$.

Definition 2.9. Two mappings A and S on fuzzy 3-metric space X are weakly commuting if and only if $M(ASu, SAu, a, b, t) \geqslant M(Au, Su, a, t)$ for all $u, a, b \in X$ and $t > 0$.

3 Main Result

Theorem 3.1. *Let $(X, M, *)$ be a complete fuzzy 3-metric space with the condition (FM-6) and let F and T be continuous mappings of X in X. Let A be a self mapping of X satisfying $\{A, F\}$ and $\{A, T\}$ are R-weakly commuting and $A(X) \subseteq F(X) \cap T(X)$,*

$$M(A^2x, A^2y, a, b, t) \geqslant \varphi \min\{M(F^2x, T^2y, a, b, t)M(F^2x, A^2x, a, b, t)M(F^2x, A^2y, a, b, t)$$
$$M(T^2x, A^2y, a, b, t)M(A^2x, T^2y, a, b, t)M(F^2x, A^2y, a, b, t)\}$$

for all $x, y, \in X$, where $\psi : [0, 1] \to [0, 1]$ is continuous function such that $\psi(t) > 1 foreach 0 \leqslant t \leqslant 1$. for $\psi(t) = 1, a, b, \in X$. the $\{x_n\}$and $\{y_n\}$ in X are such that $x_n \to x$ and $y_n \to y, \lim_{t\to\infty} M(x_n, y_n, a, b, t) \to M(x, y, a, b, t)$ for a, b, where $t > 0$. Then F, T and A have a unique common fixed point in X.

Proof: we define $\{x_n\}$ and $\{y_n\}$ such that $y_{2n} = A^2x = F^2x_{2n+1}y_{2n+1} = A^2x_{2n} = F^2x_{2n+1}$ and $y_{2n+1} = A^2x_{2n+1} = F^2x_{2n+1}$ for $n = 1, 2, \cdots$ now we shall prove $\{y_n\}$is a Cauchy sequence . Let

$$G_{2n} = M(y_ny_{n+1}, a, b, t) = M(A^2x_{2n},\ A^2x_{2n+1}, a, b, t)$$
$$\geqslant \varphi \min\{M(F^2x_{2n+1}, T^2x_{2n}, a, b, t)M(F^2x_{2n+1}, A^2x_{2n+1}, a, b, t)M(F^2x_{2n+1}, A^2, x_{2n}, a, b, t)$$
$$M(T^2x_{2n+1}A^2x_{2n}, a, b, t)M(A^2x_{2n+1}, T^2x_{2n}, a, b, t)M(F^2, x_{2n}A^2x_{2n}, a, b, t)\}$$
$$= \varphi \min\{M(y_{2n}, y_{2n-1}, a, b, t)M(y_{2n}, y_{2n+1}, a, b, t)M(y_{2n}, y_{2n}, a, b, t)$$
$$M(y_{2n-1}, y_{2n}, a, b, t)M(y_{2n+1}, y_{2n-1}, a, b, t)M(y_{2n-1}, y_{2n}, a, b, t)$$
$$= \varphi \min\{M(G_{2n-1}, G_(2n,), 1, G_{2n-1}G_(2n,), G_{2n-1}, G_{2n-1}a, b, t). \qquad \cdots\cdots (1)$$

If $G_{2n-1} \geqslant G_{2n}$ then $G_{2n} \geqslant \varphi[G_{2n-1}] > G_{2n-1}$, a contradiction therefore $G_{2n-1} \leqslant G_{2n}$. Thus $\{G_{2n}\}$ in $n \geqslant 0$ is increasing of positives real number [0,1]. Therefore we have $G_{2n} \geqslant \varphi[G_{2n-1}] > G_{2n-1}$ is increasing on [0,1] and there fore tend to limits $l_1 \leqslant 1$, it is clear $l_1 = 1$ and because if $l_1 < 1$ and, then on taking limit as $n \to \infty$ and we get $l_1 \geqslant \varphi[l_1] > l_1$, a contradiction hence $l_1 = 1$. Now for any integers $M(y_n, y_(n + m), a, b, t) \geqslant M(y_n, y_{n+1}, a, b, t/m) * \cdots * M(y_(n + m - 1), y_(n + m), a, b, t/m)* \geqslant M(y_n, y_{n+1}, a, b, t/m) * \cdots * M(y_n, y_{n+1}, a, b, t/m)$ There fore $\lim_{n\to\infty} M(y_n, y_(n + m), a, b, t) \geqslant 1 * 1 * \cdots * 1$. Because $\lim_{n\to\infty} M(y_n, y_{n+1}, a, b, t) = 1$ for $t > 0$. Thus $\{y_n\}$ is a cauchy sequence and by the completeness of X. $\{y_n\}$ converges to $u \in X$. So its subsequences $\{A^2x_{2n+1}\}, \{F^2x_{2n}\}$ and $\{AF^2x_{2n+1}\}$ also converges to same point u since $[A, K]$ is commuting so $M(A^2F^2x_{2n+1}, F^2A^2x_{2n+1}, a, b, t) \geqslant M(A^2x_{2n+1}, F^2x_{2n+1}, a, b, t)$ On taking limit as $n \to \infty, A^2F^2x_{2n+1} = F^2A^2x_{2n+1} = F^2u$, now we will prove that $F^2u = u$ First suppose that $F^2u \neq u$ then there exist $t > 0$ such $M(A^2u, u, a, b, t) < 1$ Now

$$M(A^2F^2x_{2n+1}, A^2x_{2n}, a, b, t)$$
$$\geqslant \varphi \min\{M(F^3x_{2n+1}, T^2x_{2n}, a, b, t)M(F^3x_{2n+1}, A^(2F^2)x_{2n+1}, a, b, t)M(F^3x_{2n+1}, A^2, x_{2n}, a, b, t)$$
$$M(T^2x_{2n+1}A^2x_{2n}, a, b, t)M(A^2F^2x_{2n+1}, T^2x_{2n}, a, b, t)M(F^2, x_{2n}A^2x_{2n}, a, b, t)\}.$$

This implies $M(F^2u, u, a, b, t) \geqslant \varphi \min\{M(F^2u, u, a, b, t), M(F^2u, u, a, b, t)$
$$M(F^2u, F^2u, a, b, t)M(u, u, a, b, t)M(F^2u, u, a, b, t), M(F^2u, u, a, b, t)\}.$$

Now we claim that u is a fixed point of T. Suppose it is not so then m for any $t > 0, M(u, T^2u, a, b, t) < 1$, now

$$M(A^2u, A^2T^2x_{2n}, a, b, t) \geqslant \varphi \min\{M(F^2u, T^3x_{2n}, a, b, t), M(F^2u, A^2u, a, b, t)M(F^2u, A^2T^2x_{2n}, a, b, t)$$
$$M(T^3x_{2n}, A^2T^2x_{2n}, a, b, t)M(A^2x_{2n}, T^2x_{2n}, a, b, t), M(F^2T^2x_{2n}, A^2T^2x_{2n}, a, b, t)\}$$

this implies that

$$M(u, T^2u, a, b, t) \geqslant \varphi \min\{M(u, T^2u, a, b, t), M(u, u, a, b, t), M(u, T^2u, a, b, t),$$
$$M(T^2u, T^2u, a, b, t)M(u, T^2u, a, b, t), M(T^2u, T^2u, a, b, t)\}$$

which is a contradiction there fore $T^2u = u$. hence u is a fixed point T. Is a common fixed point of T, F and A. Uniqueness suppose there is another fixed $v \neq u$ then

$$M(A^2u, A^2v, a, b, t) \geqslant \varphi \min\{M(F^2u, T^2v, a, b, t), M(F^2u, A^2u, a, b, t)M(F^2u, A^2T^2u, a, b, t)$$
$$M(T^2v, A^2w a, b, t)M(A^2u, T^2v, a, b, t), M(F^2v, A^2v, a, b, t)\}$$

this implies that

$$M(u, v, a, b, t) \geqslant \varphi \min\{M(F^2u, T^2v, a, b, t), M(u, v, a, b, t)M(u, u, a, b, t)$$
$$M(u, v, a, b, t)M(u, v, a, b, t), M(v, v, a, b, t)\}$$

This implies that $M(u, v, a, b, t) \geqslant \varphi(M(u, v, a, b, t))A$ contradiction so $u = v$ hence A, F, T have unique fixed point. □

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