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Operations on fuzzy graphs

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Abstract The present paper discuss on operations on fuzzy graphs such as union, join and composition etc.

Key Words fuzzy Graphs, subgraphs

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1 Introduction

A mathematical frame work to describe the phenomena of uncertainity in real life situation has been suggested by Zadeh in 1965. Research on the theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its applications. This ranges from traditional mathematical subjects like logic, topology, algebra, analysis etc. to pattern recognition, information theory, artificial intelligence, operations research neural networks and planning etc. [2], [8], [9], [10], [14]. Consequently, fuzzy set theory has merged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. After the pioneering work of Rosenfeld and Yeh and Bang in 1975, when some basic fuzzy graph theoretic concepts and applications have been indicated, several authors have been finding deeper results, and fuzzy analogues of many other graph theoretic concepts. This include fuzzy trees, fuzzy line graphs, operations on fuzzy graphs, automorphism of fuzzy graphs, fuzzy interval graphs, cycles an cocyles of fuzzy graphs and metric aspects in fuzzy graphs. In this paper the definition of complement of a fuzzy graphs is modified and some properties of self coelementary fuzzy graphs are studies. We show that the automorphism groups of a fuzzy graphs and its complement are identical. Also we consider the operations of union, join, composition of fuzzy graphs and prove that complement of the union of two fuzzy graphs is the join of their complements and the complement of the join of two fuzzy graphs is the union of their complements. Finally we prove that complement of the composition of two strong fuzzy graphs is the composition of their complements. Through out this paper, we assume that is reflexive and need not consider loops. Also, the underlying set V is assumed to be finite and can be chosen in any manner so as to satisfy the definition of a fuzzy graphs in all the examples.

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The operations on (crisp) graphs such as union join, Cartesian product and composition are extended to fuzzy graphs and some of their properties are studied. In the following discussions an arc between two nodes u and v is denoted by uv rather than (u, v), because in the Cartesian product of two graphs, a node of the graphs is in fact, an ordered pair.

Definition 1.1. Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with $G_1* : (V_1, E_1)$ and $G_2* : (V_2, E_2)$ with and be the union of G_1* and G_2*. Then the union of two fuzzy graphs G1 and G2 is a fuzzy graphs $G = G_1 \cup G_2 : (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ defined by

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1, \end{cases}$$

and

$$(\mu_1 \cup \mu_2)(uv) = \begin{cases} \mu_1(u) & \text{if } uv \in E_1 - E_2 \\ \mu_2(u) & \text{if } uv \in E_2 - E_1. \end{cases}$$

Definition 1.2. Consider the join $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$ of graphs where E' is the set of all arcs joining the nodes of V_1 and V_2 where we assume that $V_1 \cap V_2 = \emptyset$. Then the join of two fuzzy graphs G_1 and G_2 is a fuzzy graphs $G = G_1 + G_2 : (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ defined by $(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u), u \in V_1 \cup V_2$ and

$$(\mu_1 + \mu_2)(uv) = \begin{cases} (\mu_1 \cup \mu_2)(uv) & \text{if } uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(v) & \text{if } uv \in E'. \end{cases}$$

Note 1.3. One can easily verify that $\overline{\overline{G}} = G$.

2 Main Results

Theorem 2.1. *(i) The complement of a complete Fuzzy graphs is a complete fuzzy graphs.*

(ii) The complement of a complete fuzzy graphs is a complete fuzzy graphs.

Proof: (i) Let $G = (V, E)$ be a complete fuzzy graphs. Therefore, $\mu_{2ij}, = \mu_{1i}.\mu_{1j}$ and $\gamma_{2ij} < \gamma_{1i}.\gamma_{1j}$ for all i and j. To prove that either (a) $\overline{\mu}_{2ij} > 0$ or $\overline{\gamma}_{2ij} > 0$, (b) $\overline{\mu}_{2ij} = 0$ or $\overline{\gamma}_{2ij} = 0$. That is $\overline{\mu}_{2ij} = (\mu_{1i}.\mu_{1j}) - \mu_{2ij} = 0$ if $u_{2ij} > 0, = \mu_{1i}$ if $u_{2ij} = 0$ and $\overline{\gamma}_{2ij} = (\gamma_{1i}.\gamma_{1j}) - \gamma_{2ij}$ for every i and j, since G is a complete fuzzy graph. (ii) Proof of (ii) is the same as proof of (i). □

Theorem 2.2. *Let $G = (V, E)$ be a self-complementary fuzzy graph. Then, there exists an isomorphism $h : V \to V$ such that $\overline{\mu}_1(v_i)\mu_{1i}; \overline{\gamma}_1(v_i)\gamma_{1i}$, for every $v_i \in V$ and $\overline{\mu}_2(h(v_i), h(v_j)) = \mu_{2i}; \overline{\gamma}_2(h(v_i), h(v_j)) = \gamma_{2i};$; for every $(v_i, v_j) \in V$.*

Proof: We have We have

$$\overline{\mu}_2\left(h\left(v_i\right), h\left(v_j\right)\right) = \left(\overline{\mu}_1\left(h\left(v_i\right)\right).\overline{\mu}_1\left(h\left(v_j\right)\right)\right) - \mu_{2ij} \Rightarrow \mu_2\left(v_i, v_j\right) = \mu_{1i}.\mu_{1j} - \mu_{2ij} = \mu_{1i}.\mu_{1j} - \mu_{2ij}$$

$$\Rightarrow \sum_{i\neq j}\mu_{2ij} = \sum_{i\neq j}(\mu_{1i}.\mu_{1j}) - \sum_{i\neq j}\mu_{2ij} \Rightarrow 2\sum_{i\neq j}\mu_{2ij} = \sum_{i\neq j}\mu_{1i}.\mu_{1j} \Rightarrow \sum_{i\neq j}\mu_{2ij} = \frac{1}{2}\sum_{i\neq j}(\mu_{1i}.\mu_{1j}.)$$

And

$$\overline{\gamma}_2(h(v_i), h(v_j)) = (\overline{\gamma}_1(h(v_i)).\overline{\gamma}_1(h(v_j))) - \gamma_{2ij}$$

$$\Rightarrow \gamma_{2ij}(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij} \Rightarrow \sum_{i\neq j}\gamma_{2ij} = \sum_{i\neq j}(\gamma_{1i}.\gamma_{1j}) - \sum_{i\neq j}\gamma_{2ij}$$

$$\Rightarrow 2\sum_{i\neq j}\gamma_{2ij} = \sum_{i\neq j}(\gamma_{1i}.\gamma_{1j}) = \sum_{i\neq j}\gamma_{2ij} = \frac{1}{2}\sum_{i\neq j}(\gamma_{1i}.\gamma_{1j}.)$$

□

Theorem 2.3. *If the G is a strong fuzzy graph, then $\overline{G}$ is also strong.*

Proof. Let $uv \in E$. Then $\overline{\mu}_2(uv) = \mu_1(u).\mu_1(v) - \mu_2(uv) = \mu_1(u).\mu_1(v) = \mu_1(u).\mu_1(v) = \mu_1(u).\mu_1(v) = \mu_1(u).\mu_1(v)$, since G is strong $= 0$ and $\overline{\gamma}_2(uv) = \gamma_1(u).\gamma_1(v) - \gamma_2(uv) = \gamma_1(uv) = \gamma_1(u).\gamma_1(v) - \gamma_1(u).\gamma_1(v)$, since g is strong $= 0$. Let $uv \in E$. Then $\overline{\mu}_2(uv) = (\mu_1(u).\mu_1(v)) - \mu_2(uv) = \mu_1(u).\mu_1(v)$ and $\overline{\gamma}_2(uv) = \gamma_1(u).\gamma_1(v) - \gamma_2(uv) = \gamma_1(u).\gamma_1(v)$. □

Theorem 2.4. *Let $G_1 = \langle V_1, E_1\rangle$ and $G_2 = \langle V_2, E_2\rangle$ be two fuzzy graphs. Then (i) $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$ (ii) $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$.*

Proof: Consider the identity map $I : V_1 \cup V_2 \to V_1 \cup V_2$. To prove (i), it is enough to prove

a) (i) $\overline{\mu_1 + \mu_1'}(v_i) = \overline{\mu_1} \cup \overline{\mu_1'}(v_i)$, (ii) $\left(\overline{\gamma_1 + \gamma_1'}\right)(v_i) = \left(\overline{\gamma_1} \cup \overline{\gamma_1'}\right)(v_i)$

b) (i) $\left(\overline{\mu_2 + \mu_2'}\right)(v_i, v_j) = \left(\overline{\mu_2} \cup \overline{\mu_2'}\right)(v_i, v_j)$, (ii) $\left(\overline{\gamma_2 + \gamma_2'}\right)(v_i, v_j) = \left(\overline{\gamma_2} \cup \overline{\gamma_2'}\right)(v_i, v_j)$

(a) (i) $$\left(\overline{\mu_1 + \mu_1'}\right)(v_i) = (\mu_1 + \mu_1')(v_i) = \begin{cases} \mu_1(v_i) & if\, v_i \in V_1 \\ \mu_1'(v_i) & if\, v_i \in V_2 \end{cases} = \begin{cases} \overline{\mu_1}(v_i) & if\, v_i \in V_1 \\ \overline{\mu_1'}(v_i) & if\, v_i \in V_2 \end{cases}$$

$$= \left(\overline{\mu_1} \cup \overline{\mu_1'}\right)(v_i)$$

(ii) $$\left(\overline{\gamma_1 + \gamma_1'}\right)(v_i) = (\gamma_1 + \gamma_1')(v_i) = \begin{cases} \gamma_1(v_i) & if\, v_i \in V_1 \\ \gamma_1'(v_i) & if\, v_i \in V_2 \end{cases} = \begin{cases} \overline{\gamma_1} & if\, v_i \in V_1 \\ \overline{\gamma_1'} & if\, v_i \in V_2 \end{cases}$$

$$= \left(\overline{\gamma_1} \cup \overline{\gamma_1'}\right)(v_i),$$

b) (i) $\left(\overline{\mu_2 + \mu_2'}\right)(v_i, v_j) = (\mu_1 + \mu_1')(v_i) . (\mu_1 + \mu_1')(v_j) - (\mu_2 + \mu_2')(v_i, v_j)$

$$= \begin{cases} (\mu_1 \cup \mu_1')(v_i) . (\mu_1 \cup \mu_1')(v_i) - (\mu_2 \cup \mu_2')(v_i, v_j) & if\ (v_i, v_j) \in E_1 \cup E_2 \\ (\mu_1 \cup \mu_1')(v_i) . (\mu_1 \cup \mu_1')(v_j) - (\mu_1 \cup \mu_1')(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \mu_1(v_i).\mu_1(v_j) - \mu_2(v_i v_j) & if\ (v_i, v_j) \in E_1 \\ \mu_1'(v_i).\mu_1'(v_j) - \mu_2'(v_i, v_j) & if\ (v_i, v_j) \in E_2 \\ \mu_1(v_i).\mu_1'(v_j) - \mu_1(v_i).\mu_1(v_j) & if\ (v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \overline{\mu_2}(v_i, v_j) & if\,(v_i, v_j) \in E_1 \\ \overline{\mu_2'}(v_i, v_j) & if\,(v_i, v_j) \in E_2 = \left(\overline{\mu_2} \cup \overline{\mu_2'}\right)(v_i, v_j) \\ 0 & if\,(v_i, v_j) \in E' \end{cases}$$

(b) (ii) $\left(\overline{\gamma_2 + \gamma_2'}\right)(v_i, v_j) = (\gamma_1 + \gamma_1')(v_i) . (\gamma_1 + \gamma_1')(v_j) - (\gamma_2 + \gamma_2')(v_i, v_j)$

$$= \begin{cases} (\gamma_1 \cup \gamma_1')(v_i) . (\gamma_1 \cup \gamma_1')(v_i) - (\gamma_2 \cup \gamma_2')(v_i, v_j) & if\,(v_i, v_j) \in E_1 \cup E_2 \\ (\gamma_1 \cup \gamma_1')(v_i) . (\gamma_1 \cup \gamma_1')(v_j) - (\gamma_1 \cup \gamma_1')(v_j) & if\,(v_i, v_j) \in E' \end{cases}$$

$$= \begin{cases} \gamma_1(v_i) . \gamma_1(v_j) - \gamma_2(v_i v_j) & if\,(v_i, v_j) \in E_1 \\ \gamma_1'(v_i) . \gamma_1'(v_j) - \gamma_2'(v_i, v_j) & if\,(v_i, v_j) \in E_2 \\ \gamma_1(v_i) . \gamma_1'(v_j) - \gamma_1(v_i) . \gamma_1'(v_j) & if\,(v_i, v_j) \in E' \end{cases} = \begin{cases} \overline{\gamma_2}(v_i, v_j) & if\,(v_i, v_j) \in E_1 \\ \overline{\gamma_2'}(v_i, v_j) & if\,(v_i, v_j) \in E_2 \\ 0 & if\,(v_i, v_j) \in E' \end{cases}$$

$$= \left(\overline{\gamma_2} \cup \overline{\gamma_2'}\right)(v_i, v_j)$$

To prove (ii), it is enough to prove that

a) (i) $\left(\overline{\mu_1 \cup \mu_1'}\right)(v_i) = \left(\overline{\mu_1} + \overline{\mu_1'}\right)(v_i)$, (ii) $\left(\overline{\gamma_1 \cup \gamma_1'}\right)(v_i) = \left(\overline{\gamma_1} + \overline{\gamma_1'}\right)(v_i)$

b) (i) $\left(\overline{\mu_2 \cup \mu_2'}\right)(v_i, v_j) = \left(\overline{\mu_2} + \overline{\mu_2'}\right)(v_i, v_j)$, (ii) $\left(\overline{\gamma_2 \cup \gamma_2'}\right)(v_i, v_j) = \left(\overline{\gamma_2} + \overline{\gamma_2'}\right)(v_i, v_j)$

Consider the identity map: $I : V_1 \cup V_2 \to V_1 \cup V_2$

(a) (i) $\left(\overline{\mu_1 \cup \mu_1'}\right)(v_i) = (\mu_1 \cup \mu_1')(v_i) = \begin{cases} \mu_1(v_i) & if\, v_i \in V_1 \\ \mu_1'(v_i) & if\, v_i \in V_2 \end{cases} = \begin{cases} \overline{\mu_1}(v_i) & if\, v_i \in V_1 \\ \overline{\mu_1'}(v_i) & if\, v_i \in V_2 \end{cases}$

$$= \left(\overline{\mu_1} \cup \overline{\mu_1'}\right)(v_i) = \left(\overline{\mu_1} + \overline{\mu_1'}\right)(v_i)$$

(a) (ii) $\left(\overline{\gamma_1 \cup \gamma_1'}\right)(v_i) = (\gamma_1 \cup \gamma_1')(v_i)$

$$= \begin{cases} ll\gamma_1(v_i)\, if\, v_i \in V_1 \\ \gamma_1^{'}(v_i)\, if\, v_i \in V_2 \end{cases} = \begin{cases} ll\overline{\gamma_1}(v_i)\, if\, v_i \in V_1 \\ \overline{\gamma_1^{'}}(v_i)\, if\, v_i \in V_2 \end{cases} = \left(\overline{\gamma_1} \cup \overline{\gamma_1^{'}}\right)(v_i) = \left(\overline{\gamma_1} + \overline{\gamma_1^{'}}\right)(v_i)$$

b) (i) $\left(\overline{\mu_2 \cup \mu_2^{'}}\right)(v_i, v_j) = \left(\mu_1 \cup \mu_1^{'}\right)(v_i) . \left(\mu_1 \cup \mu_1^{'}\right)(v_j) - \left(\mu_2 \cup \mu_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} \mu_1(v_i) . \mu_1(v_j) - \mu_2(v_i, v_j)\, if\,(v_i, v_j) \in E_1 \\ \mu_1^{'}(v_i) . \mu_1^{'}(v_j) - \mu_2^{'}(v_i, v_j)\, if\,(v_i, v_j) \in E_2 \\ \mu_1(v_i) . \mu_1^{'}(v_j) - 0 if v_i \in V_1, v_j \in V_2 \end{cases} = \begin{cases} \overline{\mu_2}(v_i, v_j)\, if\,(v_i, v_j) \in E_1 \\ \overline{\mu_2^{'}}(v_i, v_j)\, if\,(v_i, v_j) \in E_2 \\ \mu_1(v_i) . \mu_1^{'}(v_j)\, if v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\mu_2} \cup \overline{\mu_2^{'}}(v_i, v_j)\, if\,(v_i, v_j) \in E_1\, or\, E_2 \\ \mu_1(v_i) . \mu_1^{'}(v_j)\, if\,(v_i, v_j) \in E' \end{cases} = \left(\overline{\mu_2} + \overline{\mu_2^{'}}\right)(v_i, v_j)$$

(b) (ii) $\left(\overline{\gamma_2 \cup \gamma_2^{'}}\right)(v_i, v_j) = \left(\gamma_1 \cup \gamma_1^{'}\right)(v_i) . \left(\gamma_1 \cup \gamma_1^{'}\right)(v_j) - \left(\gamma_2 \cup \gamma_2^{'}\right)(v_i, v_j)$

$$= \begin{cases} \gamma_1(v_i) . \gamma_1(v_j) - \gamma_2(v_i, v_j)\, if\,(v_i, v_j) \in E_1 \\ \gamma_1^{'}(v_i) . \gamma_1^{'}(v_j) - \gamma_2^{'}(v_i, v_j)\, if\,(v_i, v_j) \in E_2 \\ \gamma_1(v_i) . \gamma_1^{'}(v_j) - 0 if v_i \in V_1, v_j \in V_2 \end{cases} = \begin{cases} \overline{\gamma_2}(v_i, v_j)\, if\,(v_i, v_j) \in E_1 \\ \overline{\gamma_2^{'}}(v_i, v_j)\, if\,(v_i, v_j) \in E_2 \\ \gamma_1(v_i) . \gamma_1^{'}(v_j)\, if v_i \in V_1, v_j \in V_2 \end{cases}$$

$$= \begin{cases} \overline{\gamma_2} \cup \overline{\gamma_2'} (v_i, v_j)\, if\, (v_i, v_j) \in E_1\, or\, E_2 \\ \gamma_1 (v_i) . \gamma_1' (v_j)\, if\, (v_i, v_j) \in E' \end{cases} = \left(\overline{\gamma_2} + \overline{\gamma_2'}\right)(v_i, v_j)$$

□

Theorem 2.5. *Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two strong fuzzy graphs. Then $G_1 \circ G_2$ is strong fuzzy graphs.*

Proof: Let $G_1 \circ G_2 = G = (V, E)$ where $V = V_1 \times V_2$ and $E = \{(u, u_2)(u, v_2) : u \in V_1, u_2 v_2 \in E_2\}$ $U = \{(u_1, w)(v_1, w) : w \in V_2, u_1 v_1 \in E_1\}\, U\, \{(u_1, u_2)(v_1, v_2) : u_1 v_1 \in E_1, u_2 \neq v_2\}$

(i) $\mu_2 (u, u_2)(u, v_2) = \mu_1 (u) . \mu_2 (u_2 v_2)$

$= \mu_1 (u) . \left(\mu_1' (u_{21}) . \mu_1' (v_2)\right)$, since G_2 is strong

$= \mu_1 (u) . \mu_1' (u_2) . \mu_1 (u) . \mu_1' (v_2)$

$= \left(\mu_1 \circ \mu_1'\right)(u, u_2) . \left(\mu_1 \circ \mu_1'\right)(u, v_2)$ $\gamma_2 (u, u_2)(u, v_2) = \gamma_1 (u) . \gamma_2 (u_2 v_2)$

$= \gamma_1 (u) . \left(\gamma_1' (u_{21}) . \gamma_1' (v_2)\right)$, since G_2 is strong

$= \gamma_1 (u) . \gamma_1' (u_2) . \gamma_1 (u) . \gamma_1' (v_2)$

$= \left(\gamma_1 \circ \gamma_1'\right)(u, u_2) . \left(\gamma_1 \circ \gamma_1'\right)(u, v_2)$

(ii) $\mu_2 ((u_1, w)(v_1, w)) = \mu_1' (w) . \mu_2 (u_1, v_1)$

$= \mu_1' (w) . (\mu_1 (u_1) . \mu_1 (v_1))$, since G_1 is strong

$= \mu_1' (w) . \mu_1 (v_1) . \mu_1' (w) . \mu_1 (v_1)$

$= \left(\mu_1 \circ \mu_1'\right)(u_1, w) . \left(\mu_1 \circ \mu_1'\right)(v_1, w)$ $\gamma_1 ((u_1, w)(v_1, w)) = \gamma_1' (w) . \gamma_2 (u_1, v_1)$

$= \gamma_1' (w) . (\gamma_1 (u_1) . \gamma_1 (v_1))$, since G_1 is strong. □

References

1 R. Balakrishnan and K. Ranganathan, A Text Book of Graph Theory, Springer, 2000.

2 C. Bezdek and S. K. Pal, Fuzzy Models for Pattern Recognition, IEEE Press, 1992.

3 P. Bhattacharya. Pattern Recognition Lett. 6 (1987) 297-302.

4 P. Bhattacharya and F. Suraweera. Pattern Recognition Lett. 12 (1991) 413-20.

5 K. R. Bhutani. Pattern Recognition Lett. 9 (1989) 159-62.

6 W.L. Craine. Fuzzy Sets Systems, 68 (1994) 181-93.

7 M. Delgado, J. L. Verdegay and M. A. Vila. European J. Operations Research 22 (1985) 243-49.

8 G. J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy Logic : Theory and Applications, PHI, 1997.

9 B. Kosko. Fuzzy Thinking : The New Science of Fuzzy Logic, Hyperion, New York, 1993.

10 F. A. Lootsma. Fuzzy Logic for Planning and Decision Making, Kluwer, 1997.

11 J. N. Mordeson. Pattern Recognition Lett. 14 (1993) 381-84.

12 J. N. Mordeson and C. S. Peng. Information Sciences, 79 (1994) 159-70.

13 J.N. Mordeson and P. S. Nair. Information Sciences, 90 (1996) 39-49.

14 J. N. Mordeson and P. S. Nair. Physica Verlag (2000).

15 A. Rosenfeld and Fuzzy graphs. In fuzzy sets and their applications to Cognitive and Decision Processes, Zadeh L. A., Fu K. S., Shimura M., Eds. Academic Press, New York (1975) 77-95.

16 M. S. Sunitha and A. Vijaya Kumar. Information Sciences, 113 (1999) 293-300.

17 S. Sunitha and A. Vijaya Kumar. Some metric aspects of fuzzy graphs, In Proceedings of the Conference on Graph Connections, Allied Publishers (1999) 111-114.

18 R. T. Yeh and S. Y. Banh. Fuzzy relations, fuzzy graphs and teir applications to clustering analysis, In Fuzzy sets and their Applications to Cognitive and Decision Processes, Zadeh, L. A., Fu K. S., Shimara, M. Eds; Academic Press, New York, (1975) 125-49.

19 L. A. Zadeh, Fuzzy Sets, Inform. Control. 8 (1965) 338-53.