South Asian Journal of Mathematics 2013 , Vol. 3 (1) : 8 ∼ 21
www.sajm-online.com ISSN 2251-1512 RESEARCH ARTICLE

Strength of fuzzy cycles

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Abstract In this paper connectedness of strength k between two distinct vertices of a fuzzy graph, weight matrix of a fuzzy graph, strength of a fuzzy graph, strong path, extra strong path and strength of connectivity of a fuzzy graph are introduced. The strength of various fuzzy graphs such as paths, fuzzy graphs which are not fuzzy cycles, the complete fuzzy graph and regular fuzzy graph on a cycle are determined in terms of order of the graph. The strength of fuzzy cycles are determined in terms of order of the graph and the number of its weakest edges. We also proved that the strength of connectivity of the graph is the strength of the graph.

Key Words connectedness of strength, weight matrix, strength of the fuzzy graph, strength of connectivity

MSC 2010 05C72

1 Introduction

Fuzzy graph theory was introduced by Azriel Rosenfield in 1975. It has been growing fast and has wide application in various fields. In [5] Nagoor Gani and Radha introduced regular fuzzy graphs, total degree and totally regular graphs.

In this paper we introduce the concept of the weight matrix of a fuzzy graph and connectedness of strength k between two distinct vertices of a fuzzy graph. We associate a strength to each weight matrix and which we call as the strength of the corresponding fuzzy graph. Other new terminologies that we have introduced includes strong paths, extra strong paths and strength of connectivity of a fuzzy graph. We mainly concentrate on fuzzy graphs on cycles (i.e.fuzzy graphs with underlying crisp graph a cycle). The strength of a path, fuzzy graph which is not a fuzzy cycle, complete fuzzy graph and regular graph is determined in terms of the order of the graph. We prove that for a general fuzzy graph its strength of connectivity is its strength.

Throughout this paper only undirected fuzzy graphs are considered.

2 Applications

1. The theory of fuzzy graphs has wide application in almost all branches of science such as Chemistry, Botany, Mechanics, Communication networks etc.

Citation: Sheeba M.B, Raji Pilakkat, Strength of fuzzy cycles, South Asian J Math, 2013, 3(1), 8-21.

2. This result is useful in mobile communication. In the transmission of signals this will help to find the minimum length of the path between any two mobile towers which can transmit maximum signal strength. Let the mobile towers denote the vertices of the fuzzy graph having the maximum signal strength as its membership value and the membership value of the edges are determined by the strength of the signals between these towers. Using illustration in Section 6 we can find the minimum length of the path between any two mobile towers which can transmit maximum signal strength.
3. We can determine the maximum strength of the signals through this path.
4. It can be applied to determine the maximum flow of electric current between any two transformers.

3 Preliminaries

A *fuzzy subset* of a set V is a mapping $\sigma : V \longrightarrow [0,1]$. For any $u \in V, \sigma(u)$ is called the membership value of u in σ. A *fuzzy relation* on V is a fuzzy subset of $V \times V$. A *fuzzy relation* μ on V is a fuzzy relation on σ if $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$, $\forall u, v \in V$. A *fuzzy graph* G is a pair of functions $G(\sigma, \mu)$, where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ (i.e.a fuzzy relation μ on σ such that $\mu(uv) = \mu(vu)$, $\forall u, v \in V$). The underlying crisp graph of $G(\sigma, \mu)$ is denoted by $G^\star(V, E)$, where $E \subseteq V \times V$. Note that the crisp graph $G^\star(V, E)$ is a special case of a fuzzy graph with each vertex and edge of $G^\star(V, E)$ having membership value 1. A fuzzy graph G is *complete* if G^* is complete and $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all values $u, v \in V$, where uv denotes the edge between u and v. A *path* P of length $n-1$ is a sequence of distinct vertices $v_1 v_2 v_3 \dots v_n$ such that $\mu(v_i v_{i+1}) > 0$, $i = 1, 2, 3 \dots n-1$ and the membership value of a weakest edge is defined as its *strength*. If $v_1 = v_n$ and $n \geqslant 3$ then P is called a *cycle*. Let $G(\sigma, \mu)$ be a fuzzy graph such that $G^\star(V, E)$ is a cycle. Then G is a fuzzy cycle if and only if there exist more than one edge xy such that $\mu(xy) = \wedge\{\mu(uv)/\mu(uv) > 0\}$. The degree of a vertex v in a fuzzy graph G denoted by $d_G(v)$ is defined as $d_G(v) = \sum_{u \neq v} \mu(uv)$. If $d_G(v) = k$ for all $v \in V$, i.e if each vertex has same degree k, then G is said to be a *regular fuzzy graph* of degree k or $k-regular fuzzy graph$. This is analogous to the definition of regular graph in the theory of crisp graph.

Theorem 1. *[5] Let $G(\sigma, \mu)$ be a fuzzy graph where $G^\star(V, E)$ is an odd cycle. Then G is regular if and only if μ is a constant function*

Theorem 2. *[5] Let $G(\sigma, \mu)$ be a fuzzy graph where $G^\star(V, E)$ is an even cycle. Then G is regular if and only if either μ is a constant function or alternate edges have same membership values.*

4 Main Results

Definition 1. *Let $G(\sigma,\mu)$ be a fuzzy graph with underlying crisp graph $G^\star(V,E)$ with order n, the cardinality $|V|$ of V and size m, the cardinality $|E|$ of E. Let x and y be two distinct vertices of G. If there exists at least one path between x and y of length less than or equal to k then the connectedness of strength k between x and y is defined as the maximum of the strength of all paths between them of length less than or equal to k. Otherwise it is defined as zero.*

Throughout this paper, unless otherwise specified, G denotes the fuzzy graph $G(\sigma, \mu)$ and $G^\star$ the underlying crisp graph $G^\star(V, E)$ with $V = \{v_1, v_2, \dots v_n\}$ as the ordered set of vertices in the sense that v_1 as the first vertex, v_2 as the second vertex and so on and v_n as the last vertex and $E = \{e_1, e_2, \dots, e_m\}$ of $G^\star$ where $e_i = v_i v_{i+1}$. In this case e_i is incident with the vertex v_i.

Example 1. *In the fuzzy graph G in figure1 the connectedness of strength three between the vertices v_1 and v_5 is 0.3. Also the connectedness of strength two between the vertices v_1 and v_5 is 0.3.*

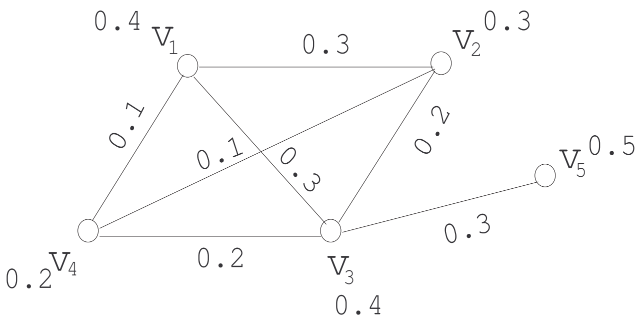

Figure 1: Fuzzy graph G

Definition 2. *Let G (σ, μ) be a fuzzy graph with underlying crisp graph $G^\star(V, E)$ with $|V| = n$ and $|E| = m$. The $n \times n$ matrix $A = (a_{ij})$ defined by*

$$a_{ij} = \begin{cases} \mu(v_i v_j) & when \quad i \neq j \\ \sigma(v_i) & when \quad i = j \end{cases}$$

is called the weight matrix associated with the graph G.

For $i \neq j$ the $(i, j)^{th}$ entry of the weight matrix of the fuzzy graph G represents the weight (membership value) of the edge $v_i v_j$ and a_{ii} represents the weight (membership value) of the vertex v_i.
By the definition of a fuzzy graph $a_{ii} \geqslant a_{ij} = a_{ji}$ for all i, j.

Example 2. *The weight matrix A of the fuzzy graph G in figure 1 is given by*

$$A = \begin{bmatrix} 0.4 & 0.3 & 0.3 & 0.1 & 0.0 \\ 0.3 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.3 & 0.2 & 0.4 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.0 & 0.5 \end{bmatrix}$$

Definition 3. *A symmetric matrix $A = (a_{ij})$ with $a_{ii} \geqslant a_{ij}$ for all i,j is called a strong diagonal matrix.*

4.1 Some Properties of the Weight Matrix associated with fuzzy graphs

1. The weight matrix A of any fuzzy graph is symmetric.
2. The sum of entries in a row or column is equal to the total degree of the corresponding vertex.
3. If A_1 and A_2 are two weight matrices which corresponds to two different labeling of the same fuzzy graph G, then for some permutation matrix P, $PA_1 = A_2P$.
4. Weight matrix of a fuzzy graph with respect to any labeling is strong diagonal.

Suppose $A = (a_{ij})$ is an $n \times n$ strong diagonal matrix with entries in $[0, 1]$. Then there exists a unique fuzzy graph (upto isomorphism) G(σ,μ) with vertices say $\{v_1, v_2 \ldots v_n\}$ such that $\sigma(v_i) = a_{ii}$ and for each pair i,j, $(i \neq j)$, $\mu(v_iv_j) = \mu(v_jv_i) = a_{ij}$.
Let $A = (a_{ij})$ and $B = (b_{ij})$ be the given strong diagonal matrices with entries in [0,1]. We define the product AB of A and B as the matrix $C = (c_{ij})$ where c_{ij} is the join of the meet of the corresponding entries of i^{th} row of A and the corresponding entries of j^{th} column of B. We define A^n, for $n \geqslant 2$, by $A^{n-1}A$.
In general the matrix multiplication defined above on the class of $n \times n$ strong diagonal matrices is not a binary operation. For example consider the matrices

$$A = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0. & 0.2 \\ 0.2 & 0.4 & 0.6 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.3 \end{bmatrix} \qquad B = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.2 \\ 0.2. & 0.4 & 0.3 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.0 \\ 0.2 & 0.1 & 0.0 & 0.5 \end{bmatrix}$$

then the product C of A and B is

$$C = \begin{bmatrix} 0.3 & 0.3 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.3 & 0.2 \\ 0.2 & 0.4 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.3 \end{bmatrix}$$

But if we restrict the class by considering only the strong diagonal $n \times n$ matrices with fixed diagonal entries, then the matrix multiplication defined above is a binary operation.

proposition 1. *Let $A = (a_{ij})$ and $B = (b_{ij})$ be two strong diagonal matrices of the same order. Suppose that $a_{ii} = b_{jj}$, for all i, j. Then their product is again a strong diagonal matrix.*

Proof. Suppose $A = (a_{ij})$ and $B = (b_{ij})$ be two strong diagonal matrices with $a_{ii} = b_{ii}$ for every i. Since A and B are strong diagonal matrices we have,
$a_{ii} \geqslant a_{ij} = a_{ji}$ and $b_{ii} \geqslant b_{ij} = b_{ji}$ for all i, j. Suppose $AB = C = (c_{ij})$ Then

$$c_{ij} = \bigvee_{k=1}^{n} (a_{ik} \wedge b_{kj}) \leqslant (a_{ii} \wedge b_{jj}) = a_{ii} = b_{jj}.$$

Thus $c_{ii} \geqslant c_{ij} \ \forall \ i, j$ □

Definition 4. *Given an $n \times n$ matrix A, the least positive integer n such that $A^n = A^i$, for all $i \geqslant n$, is called its strength.*

Definition 5. *If G is a fuzzy graph with weight matrix A, then the strength of G is defined as the strength of its weight matrix.*

proposition 2. *Suppose $A = (a_{ij})$ is the weight matrix of the fuzzy graph G with $\sigma(v_i) = a_{ii}$ and $\mu(v_i v_j) = a_{ij}$. Suppose $A^2 = (a_{ij}^{(2)})$. Then for $1 \leqslant i \neq j \leqslant n$ the $(i,j)^{th}$ entry $a_{ij}^{(2)}$ of A^2 is the connectedness of strength two between the vertices v_i and v_j and the $(i,i)^{th}$ entry $a_{ii}^{(2)}$ of A^2 is $\sigma(v_i) = a_{ii}$ for all i.*

Proof. For $i = j$, $a_{ii}^{(2)} = \bigvee_{k=1}^{n} (a_{ik} \wedge a_{ki}) = a_{ii} = \sigma(v_i)$. For $i \neq j$, $a_{ij}^{(2)} = \bigvee_{k=1}^{n} (a_{ik} \wedge a_{kj})$. Suppose that k is neither i nor j. If there is no edge between v_i and v_k or no edge between v_k and v_j then $a_{ik} \wedge a_{kj} = 0$. Otherwise it will represent the strength of the path from v_i to v_j of length two with v_k as an internal vertex. Suppose $k = i$, or $k = j$. In this case $a_{ik} \wedge a_{kj} = a_{ij} = \mu\ (v_i v_j)$. That is the strength of a path of length one between the vertices v_i and v_j. Thus $a_{ij}^{(2)}$ is the connectedness of strength two. □

The result proved in proposition [2] is in fact can be extended to any power of weight matrices as follows. If A is the weight matrix of a fuzzy graph then we denote the n^{th} power A^n of A by $(a_{ij}^{(n)})$ for $n = 2, 3, \ldots$

Theorem 3. *Suppose $A = (a_{ij})$ is the weight matrix of the fuzzy graph G. For any positive integer n, and for $i \neq j$ the $(i,j)^{th}$ entry of $A^n = (a_{ij}^{(n)})$ is the connectedness of strength n between the vertices v_i and v_j and for $i = j$ it is just $\sigma(v_i)$.*

Proof. The proof is by induction on the power n of A. Proposition [2] establishes the case $n = 2$. Suppose the result is true for $n = m$. i.e. for $i \neq j$, the $(i,j)^{th}$ entry in A^m, is the connectedness of strength m between the vertices v_i and v_j. Now consider $A^{m+1} = (a_{ij}^{(m+1)})$. First of all note that the $(i,j)^{th}$ entry of A^{m+1} is obtained by taking the join of the meet of corresponding entries of i^{th} row of A^m and the corresponding entries of the j^{th} column of A. Thus the $(i,j)^{th}$ entry $a_{ij}^{(m+1)}$ of A^{m+1} will be

$$\begin{aligned}
&= V\{a_{i1}^{(m)} \wedge a_{j1}, a_{i2}^{(m)} \wedge a_{j2} \ldots, a_{in}^{(m)} \wedge a_{jn}\} \\
&= V\{a_{i1}^{(m)} \wedge a_{1j}, a_{i2}^{(m)} \wedge a_{2j} \ldots, a_{in}^{(m)} \wedge a_{nj}\} \\
&= V_k\ \{a_{ik}^{(m)} \wedge a_{kj}\}.
\end{aligned}$$

where $a_{ik}^{(m)}$ is the connectedness of strength m between the vertices v_i and v_k. That is, it is the maximum strength of the paths from v_i to v_k of length less than or equal to m. If $a_{ik}^{(m)} \wedge a_{kj} = 0$ then either $a_{ik}^{(m)} = 0$ or $a_{kj} = 0$. If $a_{kj} = 0$ then there exists no edge in G with v_k and v_j as end vertices. Hence there is no path in G from v_i to v_j with v_k as the last but one vertex. On the other hand if $a_{ik}^{(m)} = 0$ then there exist no path in G from v_i to v_k with length less than or equal to m. Thus in either case there exists no path in G from v_i to v_j of length less than or equal to $m+1$ with v_k as a last but one vertex of that path. Thus if $a_{ik}^{(m)} \wedge a_{kj} = 0$ for all k then there exists no path between v_i and v_j of length less than or equal to $m+1$ in G and in this case $a_{ij}^{(m+1)} = 0$. Now suppose that $a_{ik}^{(m)} \wedge a_{kj} \neq 0$. In this case both $a_{ik}^{(m)}$ and a_{kj} are different from zero. The quantity $a_{ik}^{(m)} \neq 0$ implies, by induction, that there are paths

P_k in G from v_i to v_k of length less than or equal to m and $a_{ik}^{(m)}$ is the maximum strength of such paths. The quantity $a_{kj} \neq 0$ implies that the edge $v_k v_j$ exists. Thus the paths like P_k together with the edge $v_k v_j$ form paths in G of length less than or equal to $m+1$ between v_i and v_j with v_k as the last but one vertex and $a_{ik}^{(m)} \wedge a_{kj}$ is the maximum strength of the paths of length less than or equal to $m+1$ between the vertices v_i and v_j with v_k as the last but one vertex of the path. Thus $a_{ij}^{(m+1)} = V_k \ \{a_{ik}^{(m)} \wedge a_{kj}\}$ represents the maximum strength of the paths of length less than or equal to $m+1$ in G between the vertices v_i and v_j. In other words $a_{ij}^{(m+1)}$ is the connectedness of strength $m+1$ between the vertices v_i and v_j. Hence by induction the theorem follows. □

5 Strength of fuzzy graphs

In this section we study the strength of fuzzy graphs such as paths, fuzzy graphs which are not a fuzzy cycles, complete fuzzy graphs and regular graphs are determined in terms of the order of the graph.

Theorem 4. *Consider a fuzzy graph G with underlying crisp graph $G^\star$. Suppose that $G^\star$ is the path $P = v_1 v_2 v_3 \ldots v_n$. Then the strength of the graph G is the length $n-1$ of the path P.*

Proof. Consider the path $P = v_1 v_2 v_3 \ldots v_n$ of length $n-1$ and its weight matrix A. By theorem [3], the $(i,j)^{th}$ entry of A^m is the connectedness of strength m between the vertices v_i and v_j. Assume that $m > n-1$. As P being a path of length $n-1$, the maximal path in P is of length is equal to $n-1$. Hence the connectedness of strength $n-1$ and m will be the same, i.e. the $(i,j)^{th}$ entry of A^m and A^{n-1} will be the same. Hence $A^{n-1} = A^m$. But $A^{n-1} \neq A^p$ for any $p = 1, 2 \ldots n-2$. Consider the $(1, p+2)^{th}$ entry in A^p. It is the connectedness of strength p between the vertices v_1 and v_{p+2}. We know that the only path from v_1 to v_{p+2} is of length $p+1$. i.e. there is no path between v_1 and v_{p+2} of length less than or equal to p. Hence $(1, p+2)^{th}$ entry in A^p is zero. But in G there is a path between v_1 and v_{p+2} of length $p+1 \leqslant n-1$ and hence $(1, p+2)^{th}$ entry in A^{n-1} will be non-zero. Hence the theorem. □

Almost a similar result of a path will hold if the fuzzy graph is not a fuzzy cycle but the corresponding crisp graph is a cycle.

Theorem 5. *If G is not a fuzzy cycle but $G^\star$ is a cycle of length n then the strength of the graph G is $n-1$.*

Proof. Given that the crisp graph $G^\star$ is a cycle of length n and let the vertices of $G^\star$ in order be denoted by $v_1, v_2 \ldots v_n$ in the sense that for each i, $v_i v_{i+1}$ is an edge in G. As G not being a fuzzy cycle, there exists exactly one edge $v_i v_{i+1}$ such that $\mu_0 = \wedge\{\mu\ (uv)\ /\mu\ (uv) > 0\} = \mu\ (v_i v_{i+1})$. As $G^\star$ being a cycle of length n, the length of the longest path in $G^\star$ is $n-1$. Given any two distinct vertices v_i and v_j there are two paths between them. If one of the paths is of length k, then the other path is of length $n-k$. But k and $n-k$ are less than or equal to $n-1$. Thus for $i \neq j$ the $(i,j)^{th}$ entry of A^{n-1} is the maximum strength of these two paths. Since there exists only one weakest edge in G, the $(i,j)^{th}$ entry will be greater than μ_0, the membership value of the weakest edge. As there exist no path in G of length greater than $n-1$, for any $m \geqslant n$, the $(i,j)^{th}$ entry of A^m, becomes the connectedness of strength $n-1$

between the vertices v_i and v_j. Hence we have $A^m = A^{n-1}$ for all $m > n-1$. But $A^{n-1} \neq A^p$ for any $p = 1, 2 \ldots n-2$. Let v_iv_{i+1} be the weakest edge in G. Consider $(i, i+1)^{th}$ entry of A^p. Even though there are two paths between v_i and v_{i+1}, as one path is of length $n-1$ which is greater than p, the other path which is of length one will only be considered in the computation of the $(i, i+1)^{th}$ entry of A^p. As such the $(i, i+1)^{th}$ entry of A^p is the membership value $\mu(v_iv_{i+1})$ of the edge v_iv_{i+1}. However in A^{n-1}, the $(i, i+1)^{th}$ entry will be the strength of the path which does not contain the weakest edge, which is clearly, strictly greater than the membership value of the weakest edge. □

Next let us consider the case of a complete fuzzy graph.

Theorem 6. *Let G be a complete fuzzy graph. Then the strength of the graph G is one.*

Proof. As G being a complete fuzzy graph, $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all u, $v \in V$. Then for $i \neq j$, the $(i, j)^{th}$ entry of A is $\mu(v_iv_j)$. But $\mu(v_iv_j) = \sigma(v_i) \wedge \sigma(v_j)$, i.e. $\mu(v_iv_j)$ is the minimum of the membership value of $\sigma(v_i)$ and $\sigma(v_j)$. For $i \neq j$ the $(i, j)^{th}$ entry of A^2 is connectedness of strength two between the vertices v_i and v_j. As $G^\star$ being complete for any k, $v_iv_kv_j$ is a path in $G^\star$. When $k = i$ or $k = j$, it becomes the path is v_iv_j, which is of length one and its strength is $\sigma(v_i) \wedge \sigma(v_j) \ldots (1)$. Suppose that k is neither i nor j.

Case 1: Let $\sigma\ (v_k) \leqslant \sigma(v_i) \wedge \sigma(v_j)$. Then the strength of the path $v_iv_kv_j$ is $\sigma\ (v_k) \ldots (2)$. Hence from (1) and (2), $(i, j)^{th}$ entry of A^2 will be maximum of $\{\sigma(v_k),\ \sigma(v_i) \wedge\ \sigma(v_j)\}$. $= \sigma(v_i) \wedge \sigma(v_j)$, which is the $(i, j)^{th}$ entry of A.

Case 2: Let $\sigma(v_k) > \sigma(v_i) \vee \sigma(v_j)$. As the strength of the path $v_iv_kv_j$ is the minimum of $\mu(v_iv_k)$ and $\mu\ (v_kv_j)$, i.e.

$$\begin{aligned}
&= \wedge\{\sigma(v_i) \wedge \sigma(v_k), \sigma(v_k) \wedge \sigma(v_j)\} \\
&= \wedge\{\sigma(v_i) \wedge \sigma(v_k) \wedge \sigma(v_j)\} \\
&= \sigma(v_i) \wedge \sigma(v_j) \ldots (3)
\end{aligned}$$

Hence from (1) and (3) the $(i, j)^{th}$ entry of A^2 will be $\sigma(v_i) \wedge \sigma(v_j)$ which is the $(i, j)^{th}$ entry of A.

Case 3: Let $\sigma(v_i) < \sigma(v_k) < \sigma(v_j)$. In this case both the paths v_iv_j and $v_iv_kv_j$ have strength $\sigma(v_i)$. Therefore the $(i, j)^{th}$ entries of both A and A^2 are the same. This completes the proof. □

Let G be a regular fuzzy graph with crisp graph $G^\star$ a cycle of length n. As G being regular, when n is odd all edges have the same membership value and when n is even all edges have the same membership value or alternate edges have the same membership value.

Theorem 7. *Let G be a regular fuzzy graph with crisp graph $G^\star$ a cycle of length n. Then the strength of G is $[n/2]$.*

Proof. Let G be a regular fuzzy graph with the corresponding crisp graph $G^\star$ is the cycle $v_1v_2 \ldots v_nv_1$ (say).

Case 1: n odd.

As G being regular, each edge has the same membership value. As mentioned in the proof of theorem

[5] given any two vertices v_i and v_j, there are two paths between v_i and v_j . In this case both the paths have the same strength. Note that one of the paths between v_i and v_j is of length less than or equal to $[n/2]$. Thus the $(i,j)^{th}$ entry of $A^{[n/2]}$ and $A^{[n/2]+1}$ will be the same. Hence $A^{[n/2]} = A^{[n/2]+1}$.
But $A^{[n/2]} \neq A^p$ for any $p = 1, 2, \dots [n/2] - 1$.
Consider $(1, [n/2] + 1)^{th}$ entry of the matrices $A^{[n/2]}$ and A^p. As the length of the two available paths from v_1 to $v_{[n/2]+1}$ are of lengths [n/2] and [n/2]+1, the $(1, [n/2] + 1)^{th}$ entry of A^p will be zero whereas that of $A^{[n/2]}$ will be non-zero.
Case 2: n even.
In this case n must be greater than or equal to 4. As G being regular, all the edges have same membership value or alternate edges have same membership value. The proof is obvious in the first case. In the second case, for $i \neq j$, consider the $(i,j)^{th}$ entry of $A^{n/2}$ and $A^{(n/2)+1}$. Suppose there is a path of length $n/2$ between v_i and v_j. Then the other path is of length $n/2$ and these two paths have equal strength. If there is a path between v_i and v_j of length $(n/2) + 1$ then the other path is of length $(n/2) - 1$. Clearly the strength of the path of length $(n/2) - 1$ is greater than or equal to the strength of the path of length $(n/2) + 1$. Hence the $(i,j)^{th}$ entries of $A^{n/2}$ and $A^{(n/2)+1}$ are the same in this case. If there is a path between v_i and v_j of length greater than $(n/2) + 1$, the other path is of length less than $(n/2) - 1$. Thus $(i,j)^{th}$ entry in both cases will be the strength of the path of length less than (n/2)-1.Thus we can conclude that $A^{n/2} = A^{(n/2)+1}$.
But $A^{n/2} \neq A^p$ for any $p = 1, 2, \dots (n/2) - 1$.
Consider the $(1, (n/2) + 1)^{th}$ entry of the matrices $A^{n/2}$ and A^p. As in case 1 we can prove that the $(1, (n/2) + 1)^{th}$ entry of A^p is zero whereas that of $A^{n/2}$ is non-zero. Hence the theorem □

5.1 Strength of fuzzy cycles

In this section we determine the strength of fuzzy cycles in terms of its order and the number of its weakest edges. In some cases we observed that the strength of fuzzy cycles depends on the maximum length of the subpath which does not contain any weakest edges.

For a given path P we define its complementary path as follows.

Definition 6. *Given a path P in a fuzzy graph G with underlying crisp graph $G^\star$ a cycle, there exists another path Q in G with the same end vertices as that of P. The path Q is called the complementary path of P and vice versa.*

Let G be a fuzzy cycle of length n. We know that in a fuzzy cycle there are more than one weakest edge. In some case these weakest edges altogether may form a subpath and in some other case, they may not form a subpath. Suppose the weakest edges in a fuzzy cycle form a subpath. Then we have the following theorem.

Theorem 8. *In a fuzzy cycle G of length n, suppose there are l weakest edges where $l \leqslant [(n+1)/2]$. If these weakest edges form altogether a subpath, then the strength of the graph G is $n - l$.*

Proof. Let G be a fuzzy cycle of length n having l weakest edges. Given that these weakest edges form altogether a subpath. Let $v_1, v_2, \dots v_n$ denote the vertices and $e_1 = v_1v_2, e_2 = v_2v_3, \dots e_n = v_nv_1$ denote

the edges of the crisp graph $G^\star$. The fuzzy graph G being a fuzzy cycle, there exists more than one weakest edge with membership value $\mu_0 = \wedge\{\mu(uv)/\mu(uv) > 0\ \}$. As G being a cycle, we know that if there is a path of length k between v_i and v_j, then the complementary path is of length $n-k$. Without loss of generality suppose that $v_1v_2, v_2v_3, \ldots v_lv_{l+1}$ denote the l weakest edges in G.
case 1: $l < [(n+1)/2]$.
In this case we prove that $A^{n-l} = A^{n-l+1}$.
For $i \neq j$, consider the $(i,j)^{th}$ entry of A^{n-l} and A^{n-l+1}. i.e. the connectedness of strength $n-l$ and $n-l+1$ between the vertices v_i and v_j respectively in G. If there is a path of length $n-l+1$ between the vertices v_i and v_j in G, then the complementary path between v_i and v_j is of length $l-1$ which is less than $n-l+1$. Hence there are two paths of length less than or equal to $n-l+1$ between the vertices v_i and v_j. Whether the path of length $n-l+1$ contains all the weakest edges or not the $(i,j)^{th}$ entry of both A^{n-l} and A^{n-l+1} will be the strength of the path of length $l-1$. In some cases, one of the paths between the vertices v_i and v_j is of length greater than $n-l+1$. In this case the complementary path is of length less than $l-1$ and hence the $(i,j)^{th}$ entry will be strength of the complementary path in both the matrices A^{n-l} and A^{n-l+1}. In all other cases both the paths are of length less than or equal to $n-l$ in which the $(i,j)^{th}$ entry will be the maximum strength of these two paths in both A^{n-l} and A^{n-l+1}.

But $A^{n-l} \neq A^p$ for $p = 1, 2, \ldots, n-l-1$.
Consider $(1, l+1)^{th}$ entry of A^{n-l} and A^p where $p < n-l$. There is a path between v_1 and v_{l+1} of length l ($l < n-l$) containing all the weakest edges. The complementary path is of length $n-l$ which does not contain any of the weakest edges. If $l \leqslant p \leqslant n-l-1$, the $(1, l+1)^{th}$ entry of A^p will be μ_0, the membership value of the weakest edge and in all other cases its $(1, l+1)^{th}$ entry will be zero. But in A^{n-l}, the $(1, l+1)^{th}$ entry will be the strength of the path of length $n-l$, which is clearly greater than μ_0, since it does not contain any of the weakest edges.
Case 2: $l = [(n+1)/2]$.
Sub case 1: n even.
When n is even $l = n/2$. In this case we prove that $A^{n/2} = A^{(n/2)+1}$.
For $i \neq j$, consider the $(i,j)^{th}$ entry of $A^{n/2}$ and $A^{(n/2)+1}$. If there is a path between v_i and v_j of length $(n/2)+1$, then the complementary path is of length $(n/2)-1$. Hence between v_i and v_j there are two paths of length less than or equal to $(n/2)+1$. We know that there are $l = n/2$ weakest edges which form a subpath. Whether the path of length $(n/2)+1$ contains all the weakest edges or not the $(i,j)^{th}$ entry of both $A^{n/2}$ and $A^{(n/2)+1}$ will be the strength of the path of length $(n/2)-1$. In all other cases, either both the paths between v_i and v_j are of length is equal to $n/2$ or one of the path is of length less than $n/2-1$ and other is of length greater than $(n/2)+1$.In all these cases the $(i,j)^{th}$ entry will be the strength of the path of length less than or equal to $n/2$ in both $A^{n/2}$ and $A^{(n/2)+1}$.
But $A^{n/2} \neq A^p$,where $p = 1, 2, \ldots (n/2)-1$.
Consider $(1, (n/2)+1)^{th}$ entry in $A^{n/2}$ and A^p. There are two paths between v_1 and $v_{(n/2)+1}$ of length $n/2$. Hence the $(1, (n/2)+1)^{th}$ entry in $A^{n/2}$ is non-zero. But in A^p where $p \leqslant (n/2)-1$, it is zero since there is no path between v_1 and $v_{(n/2)+1}$ of length less than or equal to p.
Subcase 2: n odd.

For $i \neq j$, consider the $(i,j)^{th}$ entry of A^{n-l} and A^{n-l+1}. If there is a path between the vertices v_i and v_j of length $n-l+1(=l)$, then the other path is of length $l-1$. Hence both the paths between v_i and v_j are of length less than or equal to $n-l+1$. Whether the path of length $n-l+1$ contains all the weakest edges or not, the $(i,j)^{th}$ entry of A^{n-l} and that of A^{n-l+1} will be the strength of the path of length $l-1$. In all other cases, one of the paths P between the vertices v_i and v_j is of length greater than $n-l+1$ and hence $(i,j)^{th}$ entry will be the same in both A^{n-l} and A^{n-l+1} which is the strength of the complementary path of P of length less than $l-1$.

But $A^{n-l} \neq A^p$ for $p = 1, 2, \ldots n-l-1$.
Consider $(1, l+1)^{th}$ entry of A^{n-l} and A^p where $p < n-l(=l-1)$. In this case there is no path between v_1 and v_{l+1} of length less than or equal to p, because the two paths between v_1 and v_{l+1} are of length l and $n-l$. Hence $(1, l+1)^{th}$ entry will be zero. But in A^{n-l}, the $(1, l+1)^{th}$ entry will be the strength of the path of length $l-1$ which is clearly greater than zero.

□

Corollary. *Let G be a strong fuzzy graph with crisp graph $G^\star$ a cycle of length n. If each vertex has different membership value, then the strength of the graph is $n-2$.*

Proof. Let $v_1, v_2, \ldots v_n$ be the vertices of $G^\star$ with edges $v_1v_2, v_2v_3, \ldots v_nv_1$. G being a strong fuzzy graph $\mu(v_iv_j) = \sigma(v_i) \wedge \sigma(v_j)$ for all $v_iv_j \in E$. Given that $\sigma(v_i)$ is different for every i. Let v_k be a vertex such that $\sigma(v_k)$ has the least value. Hence the edges incident with v_k are the weakest edges. Thus in the given graph there will be only two weakest edges which form a subpath. Hence by the theorem[2] $A^{n-2} = A^{n-1}$. □

Next we consider the case where there are more than $[(n+1)/2]$ weakest edges which form a subpath.

Theorem 9. *Let G be a fuzzy cycle with crisp graph $G^\star$ a cycle of length n, having l weakest edges which form altogether a subpath where $l > [(n+1)/2]$. Then the strength of the graph is $[n/2]$.*

Proof. Case1: n even
Given that there are l weakest edges which form altogether a subpath where $l > n/2$. The length of the subpath which does not contain a weakest edge is less than or equal to $n/2 - 1$.
For $i \neq j$, consider the $(i,j)^{th}$ entry of $A^{n/2}$ and $A^{(n/2)+1}$. If there is a path between the vertices v_i and v_j of length $(n/2)+1$ then the complementary path is of length $(n/2)-1$. Hence there are two paths between v_i and v_j of length less than or equal to $(n/2)+1$. The path of length $(n/2)+1$ contains weakest edges. Therefore the $(i,j)^{th}$ entry of both the matrices will be the strength of the path of length $(n/2)-1$. When both the paths between v_i and v_j are of length $n/2$ then both the paths contains weakest edges and hence are of equal length. In all other cases one of the path between v_i and v_j is of length greater than $(n/2)+1$. Hence the $(i,j)^{th}$ entry will be the strength of the complementary path in both the matrices $A^{n/2}$ and $A^{(n/2)+1}$.
Case 2: n odd
Given there are $l > [(n+1)/2]$ weakest edges. The maximum length of the subpath which does not

contain a weakest edge is less than $[n/2]$.
For $i \neq j$, consider the $(i,j)^{th}$ entry of $A^{[n/2]}$ and $A^{[(n+1)/2]}$. If there is a path between the vertices v_i and v_j of length $[(n+1)/2]$ then the complementary path is of length $[n/2]$. l being greater than $[(n+1)/2]$, both the paths contain weakest edges and hence the $(i,j)^{th}$ entry of both the matrices will be the same. In all other cases one of the path between the vertices v_i and v_j is of length greater than $[(n+1)/2]$, and hence the $(i,j)^{th}$ entry of both the matrices will be the strength of the complementary path.

But $A^{[n/2]} \neq A^p$ for any $i = 1, 2, \ldots [n/2] - 1$.
Consider the $(1, [n/2]+1)^{th}$ entry of A^p and $A^{[n/2]}$. Whether n is odd or even the $(1, [n/2]+1)^{th}$ entry of $A^{n/2}$ is the strength of the path of length $[n/2]$ which is non-zero, whereas in the case of A^p all the paths between the vertices v_1 and $v_{[n/2]+1}$ are of length greater than p, where $p < [n/2]$. Hence $(1, [n/2]+1)^{th}$ entry will be zero in A^p. Thus $A^{[n/2]} \neq A^p$, $p = 1, 2 \ldots [n/2] - 1$. Hence the theorem. □

Now we study the strength of fuzzy cycles in which the weakest edges do not form a subpath.

Theorem 10. *Let G be a fuzzy cycle of length n. Suppose there are l weakest edges which do not altogether form a subpath. If $l > [n/2] - 1$ then the strength of the graph is $[n/2]$ and if $l = [n/2] - 1$ then the strength of the graph is $[(n+1)/2]$.*

Proof. Let G be a fuzzy cycle of length n with underlying crisp graph $G^\star$ with vertices $v_1, v_2, \ldots, v_n$ and edges $v_1v_2, v_2v_3, \ldots, v_nv_1$.
Case 1: $l > [n/2] - 1$

Suppose that there are greater than $[n/2]-1$ weakest edges which do not altogether form a subpath. Thus the maximum length of the subpath which does not contain any weakest edge is less than or equal to $[(n-1)/2]$.

Consider the $(i,j)^{th}$ entry of $A^{[n/2]}$ and $A^{[n/2]+1}$. If one of the paths between the vertices v_i and v_j is of length greater than or equal to $[n/2]+1$, then the complementary path is of length less than or equal to $[(n-1)/2]$. By the argument given above, the path of length greater than or equal to $[n/2]+1$ contains weakest edges. Thus the $(i,j)^{th}$ entry of both the matrices $A^{[n/2]}$ and $A^{[n/2]+1}$ will be the strength of the path of length less than or equal to $[(n-1)/2]$ between the vertices v_i and v_j. If both the paths between v_i and v_j are of length $n/2$ then the $(i,j)^{th}$ entry of both the matrices $A^{[n/2]}$ and $A^{[n/2]+1}$ will be the maximum strength of these two paths. Thus in all cases $A^{[n/2]} = A^{[n/2]+1}$.

But $A^{[n/2]} \neq A^p$ where $p = 1, 2, \ldots [n/2] - 1$.

Consider the $(1, [n/2]+1)^{th}$ entry of $A^{[n/2]}$ and A^p. The two paths between v_1 and $v_{[n/2]+1}$ are of lengths $[n/2]$ and $[n/2]+1$ when n is odd and both paths are of length $n/2$ when n is even. The $(1, [n/2]+1)^{th}$ entry of A^p is zero since there is no path between v_1 and $v_{[n/2]+1}$ of length less than or equal to p where $p < [n/2]$. But the $(1, [n/2]+1)^{th}$ entry of $A^{[n/2]}$ is nonzero.

Case 2: Suppose that there are $[n/2] - 1$ weakest edges. Thus the maximum length of the subpath which does not contain any weakest edge is equal to $[(n+1)/2]$. Consider the $(i,j)^{th}$ entry of $A^{[(n+1)/2]}$

and $A^{[(n+1)/2]+1}$. If there is a path between v_i and v_j of length $[(n+1)/2]+1$, then the complementary path is of length $[n/2]-1$ and vice versa. Whether the path of length $[(n+1)/2]+1$ contains all the weakest edges or not, the $(i,j)^{th}$ entry of both the matrices $A^{[(n+1)/2]}$ and $A^{[(n+1)/2]+1}$ will be the strength of the path of length $[n/2]-1$ between the vertices v_i and v_j. If there is a path between v_i and v_j of length less than $[(n+1)/2]+1$, i.e. less than or equal to $[(n+1)/2]$ then the complementary path is of length greater than $[n/2]-1$, i.e.greater than or equal to $[n/2]$. Whether the path of length greater than or equal to $[n/2]$ contains all the weakest edges or not, the $(i,j)^{th}$ entry of both the matrices $A^{[(n+1)/2]}$ and $A^{[(n+1)/2]+1}$ will be the strength of the path of length less than or equal to $[(n+1)/2]$ between the vertices v_i and v_j. Thus in all cases $A^{[(n+1)/2]} = A^{[(n+1)/2]+1}$. But $A^{[(n+1)/2]} \neq A^p$ where $p = 1, 2, \cdots, [(n+1)/2]-1$. Consider the $(1, [n/2]+1)^{th}$ entry of $A^{[(n+1)/2]}$ and A^p where $p \leqslant [(n+1)/2]-1$. Between v_1 and $v_{[n/2]+1}$ there are two paths of lengths $[n/2]$ and $[n/2]+1$ when n is odd and both the paths are of length $n/2$ when n is even. Thus the $(i,j)^{th}$ entry of A^P will be zero, since there is no path between v_1 and $v_{[n/2]+1}$ of length less than or equal to $[(n+1)/2]-1$. But the $(i,j)^{th}$ entry of $A^{[(n+1)/2]}$ is non-zero. □

When a fuzzy cycle of length n has less than $[n/2]-1$ weakest edges which do not altogether form a subpath, then the strength of the fuzzy cycle depends on the maximum length of the subpath which does not contain a weakest edge.

Theorem 11. *In a fuzzy cycle of length n suppose there are $l < [n/2]-1$ weakest edges which do not altogether form a subpath. Let s denote the maximum length of the subpath which does not contain any weakest edge. If $s \leqslant [n/2]$ then the strength of the graph is $[n/2]$ and if $s > [n/2]$ then the strength of the graph is s.*

Proof. Given that there are only less than $[n/2]-1$ weakest edges. Also given that the maximum length of the subpath which does not contain a weakest edge is s.

Case 1:$s > [n/2]$. Consider the $(i,j)^{th}$ entry of A^s and A^{s+1}. If there is a path of length $s+1 > [n/2]+1$ between the vertices v_i and v_j, then the complementary path is of length $n-s-1 < [(n+1)/2]$. As the maximum length of the path which does not contain any weakest edge is s, the path of length $s+1$ contain at least one weakest edge. Whether the path of length $s+1$ contains all the weakest edges or not, the $(i,j)^{th}$ entry of both the matrices A^s and A^{s+1} will be the strength of the path of length $n-s-1$ between the vertices v_i and v_j. If there is a path between v_i and v_j of length less than $s+1$, then the complementary path is of length greater than $n-s-1$. In this case, the $(i,j)^{th}$ entry of both the matrices A^s and A^{s+1} will be the maximum strength of the path of length less than $s+1$ between the vertices v_i and v_j, if both the paths are of length less than $s+1$. If there is a path of length greater than $s+1$ between the vertices v_i and v_j, then the complementary path is of length less than s. Thus by theorem [1], the $(i,j)^{th}$ entry of both the matrices A^s and A^{s+1} will be the strength of the path of length less than s.

But $A^s \neq A^p$ where $p = 1, 2, \ldots s-1$. Consider the $(1, s+1)^{th}$ entry of A^s and A^p where $p \leqslant s-1$. Between v_1 and v_{s+1} there are two paths of lengths s and $n-s$ respectively. Without loss of generality we can assume that the path of length s contains no weakest edges. Thus the complementary path of length $n-s$ contains all the weakest edges. Hence if $n-s \leqslant p < s$ the $(1, s+1)^{th}$ entry of A^p will be the membership value of the weakest edge and is zero otherwise. But the $(1, s+1)^{th}$ entry of A^s will be the strength of the path of length s between the vertices v_i and v_j which does not contain the weakest edges.

Case 2: $s \leqslant [n/2]$. Consider the $(i,j)^{th}$ entry of $A^{[n/2]}$ and $A^{[n/2]+1}$. If there is a path between the vertices v_i and v_j of length $[n/2]+1$, then the complementary path is of length $[(n-1)/2]$. Whether the path of length $[n/2]+1$ contains all the weakest edges or not, the $(i,j)^{th}$ entry of both the matrices $A^{[n/2]}$ and $A^{[n/2]+1}$ will be the strength of the path of length $[(n-1)/2]$ between the vertices v_i and v_j. If there is a path between v_i and v_j of length less than $[n/2]+1$, i.e. less than or equal to $[n/2]$ then the complementary path is of length greater than $[(n-1)/2]$ respectively. In this case the $(i,j)^{th}$ entry of both the matrices $A^{[n/2]}$ and $A^{[n/2]+1}$ will be the strength of the path of length less than or equal to $[(n-1)/2]$ between the vertices v_i and v_j and if both the paths are of length $[n/2]$ then the $(i,j)^{th}$ entry will be the maximum strength of these two paths. Thus in all cases $A^{[n/2]} = A^{[n/2]+1}$.

But $A^{[n/2]} \neq A^p$ where $p = 1, 2, \ldots, [n/2]-1$. Consider the $(1, [n/2]+1)^{th}$ entry of $A^{[n/2]}$ and A^p. Between v_1 and $v_{[n/2]+1}$ there are two paths of lengths $[n/2]$ and $[n/2]+1$ when n is odd and both the paths are of length $n/2$ when n is even. Thus the $(1, [n/2]+1)^{th}$ entry of A^p will be zero, since there is no path between v_1 and $v_{[n/2]+1}$ of length less than or equal to p. But the $(1, [n/2]+1)^{th}$ entry of $A^{[n/2]}$ will be nonzero which is the strength of the path of length $[n/2]$. $\square$

6 Strength of connectivity of a fuzzy graph

In this section we prove that the strength of connectivity of a fuzzy graph is same as the strength of the graph. Let us consider the following definitions.

Definition 7. *Let G be a fuzzy graph with underlying crisp graph $G^\star$. A path P is said to connect the vertices v_i and v_j of G strongly if its strength is maximum among all paths between v_i and v_j. Such paths are called strong paths.*

Given a fuzzy graph G there may exist more than one strong path between two distinct vertices.

Definition 8. *Any strong path between two distinct vertices v_i and v_j in G with minimum length is called an extra strong path between them.*

Definition 9. *The maximum length of extra strong paths between every pair of distinct vertices in G is called the strength of connectivity of the graph.*

Theorem 12. *Let G be a fuzzy graph with underlying crisp graph G^* and A its weight matrix. Let k denote the strength of connectivity of the graph G. Then the strength of G is k.*

Proof. By theorem [3] $A^k = A^m$, for any $m \geqslant k$.
We can show that $A^k \neq A^p$, for any $p = 1, 2, \ldots k-1$.
For, choose two distinct vertices v_i and v_j of G such that there exists a path of length k between them and having maximum strength. Clearly $(i,j)^{th}$ entry of A^p is less than the $(i,j)^{th}$ entry of A^k, since $p < k$. $\square$

We illustrate theorem [12] using an example.

Illustration. Consider the fuzzy graph G in figure 1. For $i \neq j$, let a_{ij} denote the maximum strength of the path between the vertices v_i and v_j and k_{ij} the minimum length of the path having maximum strength between them. For example, consider the vertices v_1 and v_2. The paths between v_1 and v_2 are v_1v_2, $v_1v_3v_2$, $v_1v_4v_2$, $v_1v_3v_4v_2$, $v_1v_4v_3v_2$ having lengths $1, 2, 2, 3, 3$ and strengths $0.3, 0.2, 0.1, 0.1, 0.1$ respectively. The maximum strength of the path between v_1 and v_2, i.e. a_{12} is 0.3, and minimum length of the path having maximum strength between v_1 and v_2 , i.e. k_{12} is 1. Similarly,

$a_{13} = 0.3$ $a_{14} = 0.2$ $a_{15} = 0.3$ $a_{21} = 0.3$ $a_{23} = 0.3$ $a_{24} = 0.2$
$a_{25} = 0.3$ $a_{31} = 0.3$ $a_{32} = 0.3$ $a_{34} = 0.2$ $a_{35} = 0.3$ $a_{41} = 0.2$
$a_{42} = 0.2$ $a_{43} = 0.2$ $a_{45} = 0.2$ $a_{51} = 0.3$ $a_{52} = 0.3$ $a_{53} = 0.3$
$a_{54} = 0.2$
$k_{13} = 1$ $k_{14} = 2$ $k_{15} = 2$ $k_{21} = 1$ $k_{23} = 2$ $k_{24} = 2$
$k_{25} = 3$ $k_{31} = 1$ $k_{32} = 2$ $k_{34} = 1$ $k_{35} = 1$ $k_{41} = 2$
$k_{42} = 2$ $k_{43} = 1$ $k_{45} = 2$ $k_{51} = 2$ $k_{52} = 3$ $k_{53} = 1$ $k_{54} = 2$

Hence $k = \bigvee_{i \neq j} k_{ij} = 3$. Thus $A^3 = A^4 = A^5 \ldots$ Here A^3 is given by the following matrix.

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.4 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.2 & 0.5 \end{bmatrix}$$

Acknowledgements The first author acknowledge the financial support by University Grants Commission of India, under Faculty Development Programme.

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