South Asian Journal of Mathematics 2012, Vol. 2 (5): 523 ∼ 534
www.sajm-online.com ISSN 2251-1512 RESEARCH ARTICLE

gsΛ closed and open functions in Topological space

Vijilius Helena Raj ①*, S. Pious Missier②

① New Horizon College Of Engineering, Outer Ring Road,Marathahalli, Bangalore India 560 103
② Department of Mathematics, V.O.Chidambaram College, Thoothukudi, Tamilnadu, India-628 008
E-mail: vijilius@gmail.com, spmissier@gmail.com

Received: 8-1-2012; Accepted: 9-20-2012 *Corresponding author

Abstract In this paper we introduce the different notions of a new class of closed and open maps called generalised semi Lambda(gsΛ) closed and generalised semi Lambda(gsΛ) open maps in topological spaces, which turns out to be weaker forms of λ closed and λ open maps. We also introduce a map called M.gsΛ closed and M.gsΛ open maps.

Key Words gsΛ closed maps and gsΛ open maps, M.gsΛ closed maps and M.gsΛ open maps

MSC 2010 54C10

1 Introduction

In 1986, Maki [4] continued the work of Levine and Dunham on generalized closed sets and closure operators by introducing the notion of Λ-sets in topological spaces. A Λ-set is a set A which is equal to its kernel(= saturated set), i.e. to the intersection of all open supersets of A.Arenas et al.[2] introduced and investigated the notion of λ-closed sets and λ-open sets by involving Λ-sets.

In 2008 M.Caldas, S.Jafari and T.Noiri [5] introduced Λ generalized closed sets(Λg, Λ-g, gΛ) and their properties.They also studied the concept of λ closed maps.In 2007 M.Caldas,S.Jafari and T.Navalagi [7] introduced the concept of λ irresolute maps.

In this paper we establish a new class of maps called gsΛ closed maps and gsΛ open maps. This new class is a super class of closed maps and λ closed maps.Here we investigate some of their fundamental properties and the connections between these maps and other existing topological maps are studied.

We also study the properties of M.gsΛ closed and M.gsΛ open maps,which are stronger than gsΛ closed and gsΛ open maps.

Throughout this paper (X,τ), (Y,σ) and (Z,η) (or simply X, Y and Z) will always denote topological spaces on which no separation axioms are assumed unless explicitly stated.Int(A),Cl(A),Int$_\lambda$A and Cl$_\lambda$A denote the interior of A, closure of A, Derived set of A, lambda interior of A and lambda closure of A respectively.

Citation: Vijilius Helena Raj, S. Pious Missier, gsΛ closed and open functions in Topological space, South Asian J Math, 2012, 2(5), 523-534.

2 Preliminary Definitions

Definition 2.1. *A subset A of a space (X,τ) is called*

1. *A semi open set [11] if A $\subset$ Cl(Int(A))*
2. *A pre open set [1]if A $\subset$ Int(Cl(A))*
3. *A regular open set [1] if A = Int(Cl(A))*

The complement sets of semi open (respectively pre open and regular open) are called semi closed sets (respectively pre closed and regular closed). The semi closure (respectively pre closure) of a subset A of X denoted by sCl(A),(pCl(A))is the intersection of all semi closed sets (pre closed sets) containing A.

A topological space(X,τ) is said to be

1. a generalized closed [12] if Cl(A) $\subset$ U, whenever A $\subset$ U and U is open in (X,τ).
2. a g* closed [9] if Cl(A) $\subset$ U,whenever A $\subset$U and U is g-open in (X,τ).
3. semi generalized closed (denoted by sg-closed) [15] if sCl(A $\subset$ U ,whenever A$\subset$U and U is semi open in (X,τ).
4. generalized semi closed (denoted by gs-closed) [15] if sCl(A)$\subset$U ,whenever A$\subset$U and U is open in (X,τ).
5. a subset A of a space (X,τ) is called λ-closed [2] if A = B$\cap$C,where B is a Λ-set and C is a closed set.
6. a subset A of (X,τ) is said to be a Λg closed set [5] if Cl(A)$\subset$U whenever A$\subset$U,where U is λopen in (X,τ).
7. a subset A of (X,τ) is said to be a Λ-g closed set [5] if Cl_λ (A) $\subset$U whenever A $\subset$U,where U is λ open in (X,τ).
8. a subset A of (X,τ) is said to be a gΛ closed set [5] if Cl_λ (A) $\subset$U whenever A$\subset$U,where U is open in (X,τ).

The complement of above closed sets are called its respective open sets

Definition 2.2. *A function f:(X,τ)$\longrightarrow$(Y,σ) is called*

1. *[15] semi open (semi closed)if f(F) is semi open(semi closed) in (Y,σ)for any open(closed) set F in (X,τ).*
2. *[10] g closed (g open) if f(F) is g closed (g open) in (Y,σ)for every closed (open) set F in (X,τ).*
3. *[5]λ closed if f(F) is λ closed in (Y,σ)for every λ closed set F of (X,τ)*
4. *[15] irresolute if for any semi open set S of (Y,σ),f^{-1}(S) is semi open in (X,τ).*

5. *[1] gc irresolute if the inverse images of g closed sets in(Y,σ) are g closed in (X,τ).*

6. *[7] λ irresolute if the inverse image of λ open sets in Y are λ open in (X,τ).*

7. *[2, 6] λ continuous if $f^{-1}(V)$ is λ open (λ closed) in $(X,\tau)$for every open (closed) set V in (Y,σ).*

Lemma 2.3. *[12] If $f{:}X\longrightarrow Y$ is continuous and closed and if B is g closed (or g open) subset of Y, then $f^{-1}(B)$is g closed (or g open) in X.*

Lemma 2.4. *[7] Let A and B be subsets of a topological space (X,τ). The following properties hold:*

1. *A is λ-closed if and only if $A = Cl_\lambda(A)$.*

2. *$Cl_\lambda(A)= \cap\{F \in \lambda\ Cl(X,\tau)/\ A \subset F\}$*

3. *$A \subset Cl_\lambda(A) \subset Cl(A)$.*

4. *If $A \subset B$, then $Cl_\lambda(A) \subset Cl_\lambda(B)$.*

5. *$Cl_\lambda(A)$ is λ-closed.*

6. *$Int_\lambda(A)$ is the largest λ open set contained in A.*

7. *A is λ open if and only if $A = Int_\lambda(A)$.*

8. *$X\setminus Int_\lambda(A) = Cl_\lambda(X \setminus A)$.*

Definition 2.5. *A space (X,τ) is called*

(i)[12]a $T_{1/2}$ Space if every g closed subset of X is closed in X.

(ii) [10]a $T\hat{g}$ Space if every $\hat{g}$ closed subset of X is closed in X.

(iii) [10] T_b space if every gs- closed subset of X is closed in X.

Definition 2.6. *[8] i)A space (X,τ) is said to be λS-space if every λ open subset of X is semi open in X. ii)A space (X,τ) is said to be λ-space if every λ closed subset of X is closed in X.*

Proposition 2.7. *[17] In a topological space (X,τ),the following properties hold:*

1. *Every closed set is $gs\Lambda$ closed.*

2. *Every open set is $gs\Lambda$ closed.*

3. *Every λclosed set is $gs\Lambda$ closed.*

4. *In T_1 space every $gs\Lambda$ closed set is λ closed.*

5. *In Partition space every $gs\Lambda$ closed set is g closed.*

6. *In a door space every subset is $gs\Lambda$ closed.*

7. *In $T_{1/2}$ space every subset is $gs\Lambda$ closed.*

Definition 2.8. *Let (X,τ) be the topological space and $A \subseteq X$. We define gsΛ closure of A (briefly gsΛ Cl(A)) to be the intersection of all gsΛ closed sets containing A and gsΛ interior of A (briefly gsΛ Int(A)) to be the union of all gsΛ open sets contained in A.*

Lemma 2.9. *Let A and B be subsets of a topological space (X,τ). The following properties hold:*

1. *$gs\Lambda Cl(A)$is the smallest gsΛ closed set containing A.*
2. *If A is gsΛ closed then $A = gs\Lambda Cl(A)$.Converse need not be true.*
3. *$A \subset gs\Lambda Cl\ (A) \subset Cl_\lambda\ (A) \subset Cl(A)$.*
4. *If $A \subset B$, then $gs\Lambda Cl(A) \subset gs\Lambda Cl(B)$.*
5. *$gs\Lambda\ Cl(gs\Lambda Cl(A)) = gs\Lambda Cl(A)$*
6. *gsΛ Int(A) is the largest gsΛ open set contained in A.*
7. *if A is gsΛ open then $A = gs\Lambda\ int(A)$.Converse need not be true.*

Proofs are obvious from the definition and properties of gsΛ closed sets and gsΛ open sets.

3 gsΛ closed map

Definition 3.1. *A map $f:(X,\tau) \longrightarrow (Y,\sigma)$ is called gsΛ closed map if the image of each closed set in X is gsΛ closed in Y.*

Theorem 3.2. *Every closed map is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ) and a function $f:(X,\tau) \longrightarrow (Y,\sigma)$ be a closed map.Hence f(F) is closed in (Y,σ).As every closed set is gsΛ closed set by proposition 2.7, we have f(F) is gsΛ closed in Y.Thus f is a gsΛ closed map. □

Converse need not be true as seen from the following example.

Example 3.3. *Let $X=Y=\{a,b,c,d,e\}$,$\tau = \{\ \phi, X, \{\ a,b\}, \{c,d\}, \{a,b,c,d\}\}$,$\sigma = \{\ \phi, Y, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$.The identity function $f:(X,\tau) \longrightarrow (Y,\sigma)$ is a gsΛ closed map but not closed map as $f(\{a,b,e\} = \{a,b,e\}$is not closed in (Y,σ).*

Theorem 3.4. *Every contra closed map is gsΛ closed map.*

Proof: As every open set is gsΛ closed set by proposition 2.7 ,the proof is obvious. □

Converse need not be true as seen from the following example.

Example 3.5. *The function defined in the example 3.3 is gsΛ closed map but not contra closed map as $A = \{a,b,e\}$ is closed in (X,τ) but f(A) is not open in (Y,σ).*

Theorem 3.6. *Every λ closed map is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ) and a function f:(X,τ)$\longrightarrow$(Y,σ) is λ closed map. Since every closed set is λ closed [2],F is λ closed in (X,τ),Since f is λ closed map f(F)is λ closed in (Y,σ) by defn 2.2. As every λ closed set is gsΛ closed set by proposition 2.7,we have f(F) is gsΛ closed. Thus f is a gsΛ closed map. □

Converse need not be true as seen from the following example.

Example 3.7. *The function defined in the example 3.3 is gsΛ closed map but not λ closed map as $A = \{a, b, e\}$ is λ closed in (X,τ) but f(A) is not λ closed in (Y,σ).*

Remark 3.8. *gsΛ closed maps and g closed maps are independent.*

Example 3.9. *Let X=Y= $\{a,b,c,d,e\}$,(X,τ)= $\{\phi,X,\{a\}, \{b,c\}, \{a,d\}, \{a,b,c\}, \{a,b,c,d\}\}$ and (Y,σ) =$\{\phi, Y,\{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\}$. The function f:(X,$\tau$) $\longrightarrow$ (Y,σ) defined with f(a)=a,f(b)= c,f(c)= b,f(d)= d,f(e)= e is a g.closed map but not gsΛ closed map, as A=($\{e\}$) is closed in (X,τ) but f(A) is not gsΛ closed in (Y,σ). Let X=Y=$\{a,b,c,d,e\}$,X,τ=$\{\phi,X,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\}\}$ and (Y,σ)= $\{\phi, Y,\{a\},\{b,c\},\{a,d\},\{a,b,c\},\{a,b,c,d\}\}$. The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a gsΛ closed map but not g closed map, as A=$\{a\}$ is closed in (X,τ) but f(A) is not g closed in (Y,σ).*

Remark 3.10. *gsΛ closed maps and semi closed maps are independent.*

Example 3.11. *Let X=Y=$\{a,b,c,d,e\}$, (X,τ)=$\{\phi,X, \{a\}, \{b,c\}, \{a,b,c\}, \{b,c,d,e\}\}$ and (Y,σ)=$\{\phi, Y, \{a\}, \{b,c\}, \{a,d\}, \{a,b,c\}, \{a,b,c,d\}\}$. The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a gsΛ closed map but not semi closed map,as A=($\{a\}$) is closed in (X,τ) but f(A) is not semi closed in (Y,σ). Let X= Y = $\{a,b,c,d,e\}$,(X,τ)=$\{\phi, X, \{a\}, \{b,c,d\}, \{a,b,c,d\}\}$ and (Y,σ)=$\{\phi, Y,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\}\}$.The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a semi closed map but not gsΛ closed map, as A=$\{e\}$ is closed in (X,τ) but f(A)=$\{e\}$ is not gsΛ closed in (Y,σ).*

Remark 3.12. *gsΛ closed maps and gs closed maps are independent.*

Remark 3.13. *gsΛ closed maps and sg closed maps are independent.*

Example 3.14. *Let X=Y=$\{a,b,c,d,e\}$,(X,τ)=$\{\phi,X,\{c\},\{d\},\{c,d\},\{c,d,e\},\{b,c,d,e\},\{a,c,d,e\}\}$ and (Y,σ)= $\{\phi, Y,\{a\},\{b,c\},\{d,e\},\{a,b,c\},\{a,d,e\},\{b,c,d,e\}\}$. The function f:(X,$\tau$) $\longrightarrow$ (Y,σ) defined as identity maps is a gs closed map but not gsΛ closed map, as A=$\{b\}$ is closed in (X,τ) but f(A) is not gsΛ closed in (Y,σ). Let X=Y=$\{a,b,c,d,e\}$,(X,τ)= $\{\phi, X, \{a\}, \{b,c,d\}, \{b,c,d,e\}\}$ and (Y,σ)=$\{\phi, Y,\{a\},\{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$. The function f:(X,$\tau$) $\longrightarrow$ (Y,σ) defined with f(a)=e,f(b)= d,f(c)= b,f(d)= c,f(e)= a is a gsΛ closed map but neither gs closed nor sg closed map, as A=$\{a,e\}$ is closed in (X,τ) but f(A)=$\{a,e\}$ is neither gs closed nor sg closed in (Y,σ).*

Theorem 3.15. *If a function f:(X,τ)$\longrightarrow$(Y,σ) is a gsΛ closed map, and (Y,σ) is a partition space then f:(X,τ)$\longrightarrow$(Y,σ) is g closed map.*

Proof: Let F be a closed set in (X,τ). Since f is gsΛ closed map, f(F)is gsΛ closed in (Y,σ). As (Y,σ) is a partition space,by proposition 2.7 we have f(F) is g closed in (Y,σ). Thus f is a g closed map. □

Theorem 3.16. *If a function f:$(X,\tau)\longrightarrow(Y,\sigma)$ is a g closed($\hat{g}$ closed and gs closed) and (Y,σ) is a $T_{1/2}$ space(resp.T$\hat{g}$space, T_b space) then f:$(X,\tau)\longrightarrow(Y,\sigma)$ is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ).Since f is g closed map ($\hat{g}$ closed, gs closed),f(F)is g closed ($\hat{g}$ closed, gs closed) in (Y,σ). As (Y,σ) is a $T_{1/2}$ space (resp.T$\hat{g}$space, T_b space), we have f(F) is closed in(Y,σ), As every closed set is gsΛ closed, we have f(F) is a gsΛ closed in Y.Thus f is a gsΛ closed map. □

Theorem 3.17. *If a map f:$(X,\tau)\longrightarrow(Y,\sigma)$ is irresolute and a λ closed map then for every gsΛ closed set B of (X,τ), f(B) is gsΛ closed set in(Y,σ).*

Proof: Let B be a gsΛ closed set of X. Let f(B)$\subseteq$U where U is a semi open set of (Y,σ), then B$\subseteq$ $f^{-1}(U)$ holds. Since f is irresolute by definition 2.2 $f^{-1}(U)$ is semi open in X. Since B is gsΛ closed, we have $Cl_\lambda(B)\subseteq f^{-1}(U)$ and hence $f(Cl_\lambda\ (B))\subseteq U$. Since f is λ closed $f(Cl_\lambda(B)$ is λ closed. Therefore we have $Cl_\lambda(f(B))\subseteq Cl_\lambda(f(Cl_\lambda(B)))=f(Cl_\lambda(B))\subseteq U$. Hence f(B) is gs$\Lambda$ closed. □

Corollary 3.18. *If a map f:$(X,\tau)\longrightarrow(Y,\sigma)$ is a gsΛ closed map and g:$(Y,\sigma)\longrightarrow(Z,\varsigma)$ is irresolute and a λ closed map then gof is a gsΛ closed map.*

Proof: Let A be a closed set in (X,τ). Then f(A) is a gsΛ closed set in (Y,σ). Since g:$(Y,\sigma)\longrightarrow(Z,\varsigma)$ is irresolute and a λ closed map by Theorem 3.17, we have g(f(A))=(gof)(A) is gsΛ closed (Z,ς). Thus gof is a gsΛ closed map. □

Theorem 3.19. *If a mapping f:$(X,\tau)\longrightarrow(Y,\sigma)$ is a gsΛ closed map then gsΛClf(A)$\subseteq$ f(Cl(A)) for every subset A of (X,τ).*

Proof: Suppose that f is gsΛ closed and A$\subseteq$ X, then by hypothesis f(Cl(A)) is gsΛ closed in (Y,σ). Hence by lemma[2.9], gsΛCl(f(Cl(A)))=f(Cl(A)). Also f(A)$\subseteq$ f(Cl(A)) and by lemma[2.9],we have gsΛCl(f(A))$\subseteq$ gsΛcl(f(Cl(A)))= f(Cl(A)). □

Converse need not be true.

Example 3.20. *Let X=Y=$\{a,b,c,d,e\}$,(X,τ)=$\{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,c,d\}\}$ and (Y,σ)=$\{\phi, Y, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\},\{a,b,c\}\}$. Define identity map in f:$(X,\tau)\longrightarrow(Y,\sigma)$. In this gs$\Lambda$Clf(A)$\subseteq$ f(cl(A)) for every subset A of (X,τ) but f is not gsΛ closed map as A=$\{e\}$ is closed in X but f(A) is not gsΛ closed in Y.*

Theorem 3.21. *A function f:$(X,\tau)\longrightarrow(Y,\sigma)$ is a gsΛ closed map if and only if for each subset S of Y and for each open set U of X containing $f^{-1}(S)$ there is a gsΛ open set V of (Y,σ) such that S $\subseteq$ V and $f^{-1}(V)\subseteq$ U.*

Proof: (Necessity)Suppose that f is a gsΛ closed set. Let S be a subset of Y and U be an open set of X such that $f^{-1}(S)\subseteq U$. Then V = Y-f(X-U) is a gsΛ open set containing S such that $f^{-1}(V)\subseteq U$.(Sufficiency) Let F be a closed set of X.Set f(F)=B and F$\subseteq$ $f^{-1}(B)$. Then, $f^{-1}(Y\text{-}B)\subseteq X\text{-}F$ and X-F is open. By hypothesis, there is a gsΛ open set V of Y such that Y-B$\subseteq$ V and $f^{-1}(V)\subseteq$ X-F. Therefore, we have F $\subseteq$ X-$f^{-1}(V)$ and hence Y-V $\subseteq$ B=f(F)$\subseteq$ f(X-$f^{-1}(V)$$\subseteq$Y-V,which implies f(F)= Y-V.Since Y-V is gsΛ closed, f(F) is gsΛ closed and thus f is a gsΛ closed map. □

Regarding the restriction f_A of a map f:$(X,\tau)\longrightarrow(Y,\sigma)$ to a subset A of (X,τ), we have the following.

Theorem 3.22. *If f:(X,τ) $\longrightarrow$ (Y,σ) is a gsΛ closed map and A is a closed set of X, then the restriction f_A :(A,τ_A)$\longrightarrow$(Y,σ) is a gsΛ closed map.*

Proof: Let B be a closed set of (A,τ_A).Then B = A$\cap$F for some closed set F of (X,τ) and so B is closed in (X,τ).(f_A)(B)= f(B) is gsΛ closed set of (Y,σ) as f is a gsΛ closed map. Hence f_A is a gsΛ closed map. □

Theorem 3.23. *Let f:(X,τ)$\longrightarrow$ (Y,σ) is a λ closed and irresolute map and if A is open and gsΛ closed set of X, then the restriction f_A :(A,τ_A)$\longrightarrow$(Y,σ) is gsΛ closed map.*

Proof: Let C be a closed set of A.Then C is gsΛ closed in A. Since A is open and gsΛ closed in (X,τ) by[17],thm[4.29] C is gsΛ closed in X. Since f is λ closed and irresolute map by theorem[3.17] f(C) is gsΛ closed set in Y. Since f(C)=f_A(C),f_A is gsΛ closed map. □

Theorem 3.24. *Let B be semi open and gsΛ closed set of (Y,σ). If a bijective map f:(X,τ)$\longrightarrow$(Y,σ) is λ closed and A = f^{-1}(B) then f_A:(A,τ_A)$\longrightarrow$(Y,σ) is gsΛ closed.*

Proof: Let F be a closed set of A. Then F = A$\cap$H for some closed set H of (X,τ).Since H is closed in(X,τ), H is also λ closed in (X,τ). Hence f(H) is λ closed in (Y,σ) as f is a λ closed map. Therefore by [[17],Thm 4.15],f(H)$\cap$B is gsΛ closed. Using (f_A)(F)=f(F)=f(A$\cap$H) = f(f^{-1}(B) $\cap$ H)= B$\cap$f(H) is gsΛ closed. Hence f_A is gsΛ closed. □

4 On Composition of maps

Theorem 4.1. *If a function f:(X,τ)$\longrightarrow$(Y,σ) is a g closed map ($\hat{g}$ closed,gs closed) and (Y,σ) is a $T_{1/2}$ space (resp.T$\hat{g}$space,and T_b) and a function g:(Y,σ)$\longrightarrow$(Z,η) is a closed map(gsΛ closed map) then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ). Since f is g closed map ($\hat{g}$ closed,gs closed) ,f(F)is g closed map ($\hat{g}$ closed,gs closed)in (Y,σ). As(Y,σ) is a $T_{1/2}$ space (resp.T$\hat{g}$space,and T_b), we have f(F) is closed in(Y,σ), Since g:(Y,σ)$\longrightarrow$(Z,η) is a closed map(gsΛ closed map) we have g(f(F))= gof(F) is a closed set(gsΛ closed set) in (Z,η). Thus gof(F) is a gsΛ closed set in (Z,η).Thus gof is a gsΛ closed map. □

The above theorem holds good even when g is contra open map,as every open set is a gsΛ closed set by proposition 2.8.

Theorem 4.2. *If a function f:(X,τ)$\longrightarrow$(Y,σ is a contra closed map,and a function g:(Y,σ)$\longrightarrow$(Z,η)is a contra open map then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ).Then f(F)is open in (Y,σ).Since g:(Y,σ)$\longrightarrow$(Z,η) is a contra open map, g(f(F))=(gof)(F) is closed set in(Z,η), which is by by proposition 2.8 gsΛ closed set. Thus gof is gsΛ closed map. □

Theorem 4.3. *If a function f:(X_1,τ)$\longrightarrow$(X_2,σ) is a contra closed map,a function g:(X_2 ,σ)$\longrightarrow$(X_3,η) is a contra open map and a function h:(X_3,η)$\longrightarrow$ (X_4,ξ) is a contra closed map then hogof:(X_1,τ)$\longrightarrow$(X_4,ξ) is gsΛ closed map.*

Proof is similar to theorem 4.2 .

Remark 4.4. *composition of gsΛ closed map is not a gsΛ closed map.*

Example 4.5. *Let X=Y=Z=*$\{a,b,c,d,e\}$*,*
$(X,\tau)=\{\ \phi,X,\{a\},\{b\},\{a,b\},\{c,d\},\{b,c,d\},\{a,c,d\},\{c,d,e\},\{a,b,c,d\},\{a,c,d,e\},\{b,c,d,e\}\}$,
$(Y,\sigma)=\{\ \phi,Y,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,c,d\}\}$*and*
$(Z,\xi)=\{\ \phi,Y,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,c,d\}\}$.
Define identity function f:$(X,\tau)\longrightarrow(Y,\sigma)$*and identity function g:*$(Y,\sigma)\longrightarrow(Z,\xi)$*.Here f and g are gsΛ closed maps but gof: X*$\longrightarrow$ *Z is not gsΛ closed as A=*$\{b\}$ *is closed in* (X,τ) *but gof(A) is not gsΛ closed in* (Z,ξ)*.*

Theorem 4.6. *Composition of closed maps is a gsΛ closed map.*

proof is clear to the readers as every closed set is gsΛ closed set.

Theorem 4.7. *If a function f:*$(X,\tau)\longrightarrow(Y,\sigma)$ *is a closed map,and a function g:*$(Y,\sigma)\longrightarrow(Z,\eta)$ *is a gsΛ closed map then gof:*$(X,\tau\ )\longrightarrow(Z,\eta)$ *is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ).Then f(F)is closed in (Y,σ).Since g:(Y,σ)$\longrightarrow$(Z,η) is a gsΛ closed map, g(f(F))=(gof)(F) is gsΛ closed set in(Z,η). Thus gof is gsΛ closed map. □

Theorem 4.8. *Let* $f_i:(X_i,\tau_i)\longrightarrow(X_{i+1},\tau_{i+1})$
(i) If $f_1,f_2,f_3,\cdots,\ f_n$ *are closed maps then the composite functions* $f_n o f_{n-1} o f_{n-2} o f_{n-3} o \cdots\ o f_1$ *is a gsΛ closed map.*
(ii) If $f_1,f_2,f_3,\ \cdots,\ f_{n-1}$ *are closed maps and* f_n *is a gsΛ closed map then the composite functions* $f_n o f_{n-1} o f_{n-2} o f_{n-3} o \cdots o f_1$ *is a gsΛ closed map.*

Proof: Since every closed (open) set is a gsΛ closed set, the proofs follows. □

Theorem 4.9. *Let f:*$(X,\tau)\longrightarrow(Y,\sigma)$ *and g:*$(Y,\sigma)\longrightarrow(Z,\eta)$*be two mappings such that gof:*$(X,\tau\ )\longrightarrow(Z,\eta)$ *is closed map.Then*
(i) if f is continuous and surjective then g is gsΛ closed map.
(ii) if g is λ *irresolute and injective then f is gsΛ closed map.*
(iii)if g is λ *continuous and injective then f is gsΛ closed map.*
(iv)if f is λ *continuous, (g continuous,* $\hat{g}$ *continuous),surjective and* (X,τ) *is a* λ *space(*$T_{1/2}$ *space,*$T\hat{g}$ *space) then g is gsΛ closed map.*

Proof:
(i) Let A be a closed set in (Y,σ). Since f is continuous f^{-1}(A) is closed in (X,τ). Since gof is closed map and f is surjective we have (gof)(f^{-1})(A) = g(A) is closed in (Z,η),which is by [thm 3.5,[17]] gsΛ closed.Thus g is a gsΛ closed map.
(ii)Let A be a closed set in (X,τ). Since gof:(X,τ)$\longrightarrow$(Z,η) is closed map gof(A) is closed in (Z,η),which is also λ closed. Since g is λ irresolute and injective we have g^{-1}(gof)(A)= f(A) is λ closed in (Y,σ), which is also gsΛ closed set. Thus f is a gsΛ closed map.

(iii) Let A be a closed set in (X,τ). Since gof:(X,τ)$\longrightarrow$(Z,η) is closed map gof(A) is closed in (Z,η). Since g is λ continuous and injective we have g^{-1}(gof)(A)= f(A) is λ closed in (Y,σ), which is also gsΛ closed set. Thus f is a gsΛ closed map.

(iv) Let A be a closed set in (Y,σ). Since f is λ continuous (g continuous,$\hat{g}$ continuous) by definition 2.7, f^{-1}(A) is λ closed in (X,τ)(g closed ,$\hat{g}$ closed),which is a closed set as (X,τ) is a λ space (T$_{1/2}$ space,T$\hat{g}$ space).Since gof is a closed map and f is surjective we have (gof)(f^{-1})(A) = g(A) is closed in (Z,η),which is by [thm 3.5,[17]] gsΛ closed.Thus g is a gsΛ closed map. □

5 M.gsΛ closed map

Definition 5.1. *A map f:(X,τ)$\longrightarrow$(Y,σ) is called M.gsΛ closed map if the image of each gsΛ closed set in X is a gsΛ closed in Y.*

Theorem 5.2. *If a map f:(X,τ)$\longrightarrow$(Y,σ) is irresolute and a λ closed map then f is a M.gsΛ closed map.*

Theorem 5.3. *Every M.gsΛ closed map is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ)and a function f:(X,τ)$\longrightarrow$(Y,σ) be a M gsΛ closed map.F is gsΛ closed in (X,τ) as every closed set is gsΛ closed.Hence f(F) is gsΛ in(Y,σ).Thus f is gsΛ closed map. □

Remark 5.4. *M.gsΛ closed map and λ closed maps are independent concepts as seen from the following example.*

Example 5.5. *Let X=Y=$\{a,b,c,d,e\}$, (X,τ)=$\{$ ϕ, X,$\{a\}$, $\{b,c\}$, $\{a,b,c\}$, $\{b,c,d,e\}\}$ and (Y,σ)=$\{$ ϕ, Y, $\{a\}$, $\{b,c\}$, $\{d,e\}$, $\{a,b,c\}$, $\{a,d,e\}$, $\{b,c,d,e\}\}$. The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a λ closed map but not M.gsΛ closed map, as A=($\{b,d,e\}$) is gsΛclosed in (X,τ) but f(A) is not gsΛ closed in (Y,σ). Let X=Y=$\{a,b,c,d\}$,X,τ=$\{$ ϕ,X,$\{a\}$,$\{b\}$,$\{a,b\}$,$\{a,b,c,\}\}$ and (Y,σ)=$\{$ ϕ,Y,$\{a\}$,$\{b.c\}$,$\{a,b,c\}\}$. The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a M.gsΛ closed map but not λ closed map, as A=$\{b\}$ is λ closed in (X,τ) but f(A) is not λ closed in (Y,σ).*

Theorem 5.6. *If a function f:(X,τ)$\longrightarrow$(Y,σ)is a M.gsΛ closed map, and (Y,σ) is a T_1 space then f:(X,τ) $\longrightarrow$(Y,σ) is λ closed map.*

Proof: Let F be a λ closed set in (X,τ).Then F is also a gsΛ closed set (X,τ). Since f is M.gsΛ closed map, f(F) is gsΛ closed in (Y,σ). As (Y,σ) is a T$_1$ space, we have f(F) is λ closed[17]. Thus f is a λ closed map. □

Theorem 5.7. *If a function f:(X,τ)$\longrightarrow$(Y,σ)is a λ closed map, and (X,τ) is a T_1 space then f:(X,τ) $\longrightarrow$(Y,σ) is M.gs Λ closed map.*

Proof: Let F be a gsΛ closed set in (X,τ). Then F is also a λ closed set, as (X,τ) is a T$_1$ space. Since f is λ closed map,f(F)is λ closed in (y,σ) which is also by preposition 2.7 gsλ closed. Thus f is a M.gs Λ closed map. □

Theorem 5.8. *If a function f:(X,τ)$\longrightarrow$(Y,σ)be a closed map,and a function g:(Y,σ)$\longrightarrow$(Z,η)be a M.gsΛ closed map then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ).Then f(F)is closed in (Y,σ).By preposition 2.7,f(F) is gsΛ closed in(Y,σ).Since g:(Y,σ)$\longrightarrow$(Z,η) is a M.gsΛ closed map, g(f(F))=(gof)(F) is gsΛ closed.Thus gof is gsΛ closed map. □

Theorem 5.9. *If a function f:(X,τ)$\longrightarrow$(Y,σ)be a gsΛ closed map,and a function g:(Y,σ)$\longrightarrow$(Z,η)be a M.gsΛ closed map then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ closed map.*

Proof: Let F be a closed set in (X,τ).Then f(F)is gsΛ closed in (Y,σ).Since g:(Y,σ)$\longrightarrow$(Z,η) is a M.gsΛ closed map, g(f(F))=(gof)(F) is gsΛ closed in(Z,η).Thus gof is gsΛ closed map. □

Theorem 5.10. *If a function f:(X,τ)$\longrightarrow$(Y,σ)be a M.gsΛ closed map,and (Y,σ) is a partition space then f:(X,τ)$\longrightarrow$(Y,σ) is g closed map.*

The proof follows as in a partition space every gsΛ closed set is g closed by proposition 2.8

Theorem 5.11. *If a function f:(X,τ)$\longrightarrow$(Y,σ) is a M.gsΛ closed map then gsΛClf(A) $\subseteq$ f(gsΛCl(A))for every subset A of (X,τ).*

Proof: Similar to theorem 3.21. □

Theorem 5.12. *(iii) If f_1, f_2, f_3,$\cdots$, f_{n-1} are closed maps and f_n is a M. gsΛ closed map then the composite functions $f_n \circ f_{n-1} \circ f_{n-2} \circ f_{n-3} \circ \cdots \circ f_1$ is a gsΛ closed map.*
(iv) If f_1 is a closed map and f_2, f_3, $\cdots$, f_n are M- gsΛ closed maps then the composite functions $f_n \circ f_{n-1} \circ f_{n-2} \circ f_{n-3} \circ \cdots \circ f_1$ is a gsΛ closed map.
(v) If f_1 is a contra closed map and f_2,f_3,$\cdots$, f_n are M-gsΛ closed maps then the composite functions $f_n \circ f_{n-1} \circ f_{n-2} \circ f_{n-3} \circ \cdots \circ f_1$ is a gsΛ closed map.

Proof is clear to the readers.

Definition 5.13. *A map f:(X,τ)$\longrightarrow$(Y,σ) is called M.gsΛ open map if the image of each gsΛ open set in X is a gsΛ open in Y.*

Analogous to M.gsΛ closed map we can also prove results on M.gsΛ open map.

6 gsΛ open map

Definition 6.1. *A map f:(X,τ)$\longrightarrow$(Y,σ) is a gsΛ open map if the image f(A) is gsΛ open in (Y,σ) for every open set A in (X,τ).*

Theorem 6.2. *Every open map (closed) is gsΛ open map.*

Proof: Let F be a open set in (X,τ) and a function f:(X,τ)$\longrightarrow$(Y,σ) be a open map. Hence f(F) is open in (Y,σ). As every open set(closed) is gsΛ open set by prep 2.7, we have f(F) gsΛ open. Thus f is gsΛ open map. □

Converse need not be true as seen from the following example.

Example 6.3. *Let X=Y=$\{a,b,c,d,e\}$,τ= $\{\{a,b\},\{c,d\},\{a,b,c,d\}\}$,$\sigma$ = $\{\{a\},\{b,c,d\},\{a,b,c,d\}\}$. The identity function function f:(X,τ) $\longrightarrow$ (Y,σ) is a gsΛ open map but not open map as f($\{a,b\}$=$\{a,b\}$is not open in (Y,σ).*

Theorem 6.4. *Every contra open map is gsΛ open map.*

Proof: Let F be a open set in (X,τ)and a function f:(X,τ)$\longrightarrow$(Y,σ) be a contra open map.Hence f(F) is closed in(Y,σ).As every closed set is gsΛ open set by prep 2.7,we have f(F) gsΛ open.Thus f is gsΛ open map. □

Converse need not be true as seen from the following example.

Example 6.5. *Let X=Y=$\{a,b,c,d,e\}$,(X,τ)= $\{\{a,b\},\{c,d\},\{a,b,c,d\}\}$,
(Y,σ) = $\{\{a\},\{b,c,d\},\{a,b,c,d\},\{b,c,d,e\}\}$. The identity function f:(X,τ) $\longrightarrow$ (Y,σ) is a gsΛ open map but not contra open map as A=($\{a,b\}$ is open in (X,τ) but f(A) is not closed in (Y,σ).*

Theorem 6.6. *A bijection f:(X,τ)$\longrightarrow$(Y,σ) is gsΛ closed if and only if f is gsΛ open .*

Proof: Let a bijection f:(X,τ)$\longrightarrow$(Y,σ) is gsΛ closed and A be a open set in (X,τ),then A^c is closed in (X,τ). By assumption f(A^c)=f(A)^c is gsΛ closed in Y which implies f(A) is gsΛ open. Thus f is gsΛ open. Converse can be proved as similar to above. □

Theorem 6.7. *If a function f:(X,τ)$\longrightarrow$(Y,σ) is a gsΛ open map then f(int(A))$\subseteq$ gsΛ(int f(A)) for every subset A of (X,τ).*

Proof: Suppose that f is gsΛ open and A$\subseteq$ X. Since int(A) is open in X, f(int(A)) is gsΛ open in (Y,σ). We have f(int(A))$\subseteq$ f(A).Hence f(int(A)$\subseteq$ gsΛ (int(f(A))). □

Remark 6.8. *Composition of gsΛ open maps are not gsΛ open map.*

Example 6.9. *In Example 4.5 f and g are gsΛ open maps but gof: X$\longrightarrow$ Z is not gsΛ open as A=$\{a,c,d,e\}$ is open in (X,τ) but gof(A) is not gsΛ open in Z.*

Theorem 6.10. *Composition of open maps are gsΛ open map.*

Proof: Proof is obvious as every open set in gsΛ open map. □

Theorem 6.11. *i)If a function f:(X,τ)$\longrightarrow$(Y,σ) is a contra open map,and a function g:(Y,σ)$\longrightarrow$(Z,η)is a contra closed map then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ open map.*
ii)If a function f:(X,τ)$\longrightarrow$(Y,σ) is a open map,and a function g:(Y,σ)$\longrightarrow$(Z,η)is a gsΛ open map then gof:(X,τ)$\longrightarrow$(Z,η) is gsΛ open map.

Proof: Proof is obvious as every open set in gsΛ open. □

Theorem 6.12. *A function f:(X,τ)$\longrightarrow$(Y,σ) is gsΛ open if and only if for any subset B of Y and for any closed set S containing f^{-1}(B), there exist a gsΛ closed set A of Y containing B such that f^{-1}(A)$\subset$ S.*

Theorem 6.13. *A function f:$(X,\tau)\longrightarrow(Y,\sigma)$ is gsΛ open if and only if f^{-1}(gsΛ Cl(B))$\subset$Cl(f^{-1}(B)) for every subset B of (Y,σ).*

Proof: Suppose that f is gsΛ open.Then for any B$\subset$ Y,f^{-1}(B)$\subset$Cl(f^{-1}(B)). By theorem 6.12, there exist a gsΛ closed set A of Y such that B$\subset$A and f^{-1}(A)$\subset$Cl(f^{-1}(B)). Now we have by lemma 2.9 gsΛCl(B)$\subset$ gsΛCl(A)=A, since A is a gsΛ closed set.Hence we have f^{-1}(gsΛCl(B))$\subset f^{-1}$(A)$\subset$ Cl(f^{-1}(B). Thus proved. Conversely, Let S be any subset of (Y,σ) and F be any closed set containing f^{-1}(S).Put A = gsΛ Cl(S).Then A is gsΛ closed set and S$\subset$A.By assumption,f^{-1}(A)=f^{-1}(gsΛCl(S))$\subset$Cl(f^{-1}(S))$\subset$F, and therefore by theorem 6.12 f is gsΛ open. □

References

1 Arockiarani, K.Balachandranand J. Dontchev Some Characterizations of gp-irresolute and gp-continuous maps between topological spaces,*Mem.Fac.Sci.Kochi Univ.(Math)*20(1999),93-104.

2 Francisco G Arenas, Julian Dontchev and Maxmillian Ganster, On λ-sets and the dual of generalized continuity,*Question answers GEN.Topology*15(1997)3-13

3 Jin Han Park,Yong Beom Park and Bu Young Lee, On gp closed sets and pre gp continuous functions, *Indian. J. pure appl. Math* vol 3(1) :3-12, January 2002

4 H.Maki ,Generalised Λ-sets and the associated closure operator, *The special issue in Commemoration Of Prof.Kazusada IKEDS Retirement*,1 oct (1986),139-146

5 M.Caldas, S.Jafari and T.Noiri ,On Λ-generalised closed sets in topological spaces,*Acta Math. Hungar*,118(4)(2008),337-343

6 M.Caldas,and Jafari, S,On some low separation axioms via λ open and λ closure operator.*Rend.Circ.Mat.Di.Palermo* 54,2 (2005),195-208

7 M.Caldas,S.Jafari and T.Navalgi,More on λ-closed sets in topological spaces, *Revista Colombina de Mathematics*,Vol 41 (2007) 2,355-369

8 M.Caldas,Ekici,S.Jafari and Noiri,S,On the class of contra λ continuous functions.*Ann.Univ.Sci.Budapest Sec Math* 49(2006),75-86.

9 M.K.R.S.VeeraKumar ,gs- Closed Sets in topological space,*Mem.Fac, Sci.Kohi Univ.(Math)*,21(2000), 1-19

10 M.K.R.S Veera Kumar., On $\hat{g}$ closed sets in topological spaces,*Bull Alahabad.Soc*18(2003)99-112

11 N.Levine,Semi-open sets ans semi continuity in topological spaces,*Amer.Math.Monthly*,70 (1963),36-41.

12 N.Levine,Generalized closed sets in topology,*Rend.Circ.Mat.Palermo*,19(1970),89- 96.

13 Parithosh Bhattacharyya AND B.K.Lahiri On semi generalized closed sets in topology, *Indian Journal of Mathematics*, Vol 29(1987)375-382

14 P.Sundaram,H.Maki and K.Balachandran Semi-Generalised Continuous Maps and Semi-$T^{1/2}$ spaces,*Bulletine of Fukuoka University of Education*,vol,40,part III, 33-40(1991)

15 R.Devi,H.Maki and K.Balachandran, semi generalized closed maps and generalized semi closed maps, *Mem.Fac.Kochi.Univ.ser.A(Math)*,14(1993)

16 S.S.Benchalli $g^{\star}$-Pre Regular and $g^{\star}$-pre Normal spaces,*international Mathamatical Forum*,4,2009,no.48,2399-2408

17 S.P.Missier,Vijilius H Raj gsΛ closed sets in topological spaces *to be communicated*