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Conversion of the Riemann prime number formula

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Abstract In 1859, Riemann proposed a distribution of prime numbers formula. This formula is a conjecture of Riemann. Corresponding step function of the distribution of prime numbers, and thus on the Riemann primes distribution formula to convert, the Riemann formula conversion theorem, thus proving the the Riemann prime number distribution formula.

Key Words prime number, conversion, calculation, control

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1 Introduction

In 1900, a the mathematical giants par with the Gauss and Riemann, German mathematician David Hilbert listed in the speech Twenty-three mathematical problems have far-reaching significance, Riemann conjecture was listed as the eighth question from Riemann Hypothesis to be watched by the entire mathematical community problem. Riemann guess what is it?

In 1737, the famous mathematician Leonhard Euler published a landmark formula [6]

$$\sum_{n=1}^{\infty} = \prod_{p}(1 - \frac{1}{p^s})^{-1}, s > 1,$$

Where n is a positive integer. p is a prime number. s is real number. This formula is called the: Euler product formula. Began a theoretical study of the distribution of prime numbers. In 1798, French mathematician Legendre proposed for the first time a prime formula [5]

$$\pi(x) \approx \frac{x}{\ln x - 1.08366},$$

Here $\pi(x)$ indicates the number of prime numbers not greater than x. This formula: average distribution density of primes not greater than x 1/lnx Although the Legendre no given prove, however, they found the right way to study the distribution of prime numbers.

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1849, Germany math masters F. Gauss letter to Encke said, between 1792-1793, found by calculating: For large average distribution density of primes near x [6]

$$\frac{1}{\ln x},$$

Gauss guess, although it did not give proof.However, the vaguely saw the door of the subtleties of the distribution of prime numbers. 1859, the outstanding German mathematician Riemann real number s of the Euler product formula extended to the complex, published Riemann ζ function [1], [2], [5], [6]:

$$\zeta(s) = \Sigma_{n=1}^{\infty}\frac{1}{n^s},\ Re(s) > 1,$$

Riemann ζ The function can be expressed as algebraic form

$$\zeta(s) = 2\Gamma(1-s)(2\pi)^{s-1}sin(\pi s/2)\zeta(1-s),$$

When $s = -2n, \zeta(s) = 0$, called $\zeta(s)$ to zero, such is obviously zero.Such obvious called Zero: trivial zeros.Addition to the obvious zero, called: non-trivial zeros. In 1859, Riemann guess [6]: All Riemann ζ function non-trivial zeros in $Re(s) = 1/2$. This is the famous Riemann conjecture. Then non-trivial zeros of what the problem is it? Is a prime number distribution problem. If proof of the Riemann conjecture is true, then the distribution of prime numbers is ordered. If the proof is a function with $\pi(x)$ are equal, then Riemann Hypothesis definitely established. However, it is to be found with a $\pi(x)$ equal to the theorem, is extremely difficult. In 1894, the mathematician Von Mangoldt prove

$$J(x) \sim \Psi(x),$$

here $J(x)$ is a special step function. The secret of the distribution of prime numbers in this particular step function. Proof and convert the Von Mangoldt $J(x)$, the mathematicians was able to prove the prime number theorem. In 1896, mathematician Hadamard and De la Valle'e Poussin Esoteric, based on the equivalent function $J(x)$, the use of esoteric theory of entire functions independently proved the prime number theorem [3], [5]:

$$\pi(x) \sim \frac{x}{\ln x}, (x \to \infty).$$

In 1949, mathematician Selberg and Erd?s Using, with elementary methods to prove the prime number theorem. Then mathematicians Fogel thorough elementary methods to prove the prime number theorem [2], [5]:

$$\pi(x) \sim \frac{x}{\sum_{n \leqslant x}\frac{1}{n}}, (x \to \infty).$$

Mathematicians think, found a clever and useful elementary, relative to the discovery of a profound relationship is much more difficult. The prime number theorem elementary proof is so difficult to find. Selberg and Erd?s work, therefore occupies a significant position in the number theory. 2009, Dan Liu found the primes distribution of elementary transformation, Huazhong University of Science and Technology, Professor Lu Yuanhong help, prove

$$\pi(x) = s(x) = \frac{\varepsilon x}{\ln(\lambda)}\sum_{n=1}^{a}\frac{\mu(\lambda^n)}{\lambda^n}, (x \to \infty).$$

This is the function of a $\pi(x)$ are equal. It can prove that the Riemann primes the distribution formula and Gauss's guess.

2 Riemann primes distribution formula

Let a positive integer n, a well-established distribution formula [3],[4],[6]:

$$\pi(x) = \Sigma_n \frac{\mu(n)}{n} J(x^{1/n}) \tag{1}$$

$$J(x) = Li(x) - \Sigma_{Im\rho>0}[Li(x^{\rho}) + Li(x^{1-\rho})] + \int_x^{\infty} \frac{dt}{t(t^2-1)\ln t} - \ln 2. \tag{2}$$

$$\Psi(x) = x - \sum_{\rho} \frac{x^{\rho}}{\rho} - \frac{1}{2}\ln(1 - \frac{1}{x^2}) - \ln(2\pi).$$

$$\Psi(x) = x + O(\frac{x}{\ln^A x}), x \geqslant 2,$$

Which (1) the distribution formula for the Riemann primes. $J(x)$ For a particular step function, and $\Psi(x)$ Are equivalent. $\mu(n)$ Called M?bius conversion. Defined as follows:

$\mu(1) = 1$

$\mu(n) = (-1)^k$, $n = p_1p_2\cdots p_k$, p_i prime number

$\mu(n) = 0$, the rest n.

The degree of accuracy of the formula of the distribution of prime numbers, depends on the degree of offset Distribution of Zeros. Need to be considered a prime factor of n. And a $Li(x)$ function is also not completely clear. Calculation is very complex. This is the main conclusions Riermann in 1859. This conclusion contains two conjectures, need to prove: 1, With non-trivial zeros distribution related items:

$$\Sigma_{Im(\rho)>0}[Li(x^{\rho}) + Li(x^{1-\rho})],$$

Proves all Riemann η function non-trivial zeros in $Re(s) = 1/2$, those related to the Distribution of Zeros items to maximize cancel each other out, the distribution of prime numbers is ordered. Prove the close contact of the $Li(x)$ and $\pi(x)$. Is to prove Gauss's guess: the large primes near x average distribution density $1/\ln x$of. Let Riermann conjecture is true, ignore the secondary of the zero maximize offset. By (1), we get:

$$\pi(x) = Li(x) - \frac{Li(x^{1/2})}{2} - \frac{Li(x^{1/3})}{3} + \frac{Li(x^{1/4})}{2\times 2} + \cdots + (-1)^k \frac{Li(x^{1/k})}{p_1p_2\cdots p_k}$$

Obviously:

$$\frac{Li(x^{1/2})}{2} > -\frac{Li(x^{1/3})}{3} + \frac{Li(x^{1/4})}{2\times 2} + \cdots + (-1)^k \frac{Li(x^{1/k})}{p_1p_2\cdots p_k},$$

get:

$$Li(x) - Li(x^{1/2}) < \pi(x) < Li(x) + Li(x^{1/2}).$$

Wish expressed as:

$$Li(x) - 2\pi(x^{1/2}) < \pi(x) + 2\pi(x^{1/2}) \tag{2}$$

This (2) is the ideal form Riermann conjecture.

3 Mertens theorem

In 1874, Mertens proof

$$\sum_{p\leqslant x}\frac{1}{p}=\ln\ln x+A_1+O(\frac{1}{\ln x}) \tag{3}$$

(3) Mertens theorem. The coefficient of real numbers c (x), obtained by (3):

$$\sum_{p\leqslant x}\frac{1}{p}=\ln\ln x+A_1+c(x) \tag{4}$$

By (3), (4), it is easy to confirm that, for sufficiently large $x, c(x)$ is infinitely small and can be ignored. Mertens theorem and Euler product formula are well-known the primes distribution function. Although not explicitly indicated $\pi(x)$, after all, is obtained by proving.

4 the prime number theorem

In 1849, the German mathematician Gauss through a statistical guess, for large x primes near average distribution density [6]:

$$\frac{1}{\ln x}$$

get

$$Li(x)=\int_2^x\frac{du}{\ln u}, eqno(5)$$

Integration by parts, get:

$$Li(x)=\frac{1}{\ln x}+\frac{1!}{\ln^2 x}+\cdots+\frac{(k-1)!}{\ln^k x}, k\leqslant\ln x,$$

Mathematicians to prove [5]:

$$\pi(x)\sim Li(x),(x\to\infty),k\leqslant\ln x.$$

This (5) is called: the prime number theorem. The proof of the prime number theorem, also proved that the density of the average distribution of prime numbers not greater than x.

5 Riemann formula conversion Theorem

Step function defined as [6]

$$J(x)=\sum_{n=1}^{x}\frac{\pi(x^{\frac{1}{n}})}{n},$$

The integer n of the step function can be converted into λ^n positive real number, in order to reduce the ladder span. Maximize the offset and non-trivial zeros of various. In order to ensure the conversion function with $\pi(x)/x$ equal. Let x be a great, Conversion function is equivalent to [7], [8]

$$\sum_{n=1}^{x}\frac{u(x/\lambda^n)}{\lambda^n}\sim\frac{1}{x}\sum_{n=1}^{x}\frac{\pi(x^{1/n})}{n},$$

Can be expressed as:

$$\frac{\mu(x/\lambda)}{\lambda}-\frac{\mu(x/\lambda^2)}{\lambda}+\frac{\mu(x/\lambda^2)}{\lambda^2}-\frac{\mu(x/\lambda^3)}{\lambda^2}+\cdots+\frac{\mu(x/\lambda^n)}{\lambda^n}-\frac{\mu(x/\lambda^{n+1})}{\lambda^n}$$

Let x be a great, $\lambda>1$, Positive integer a, get

$$\begin{aligned}\pi(x)&=s(x)-s(\sqrt{x})+\pi(\sqrt{x})\\ s(x)&=\frac{x(\lambda-1)}{\ln x}\Sigma_{n=1}^{a}\frac{\mu(\lambda^n)}{\lambda^n},\\ s(\sqrt{x})&=\frac{x(\lambda-1)}{\ln x}\Sigma_{n=a/2+1}^{a}\frac{\mu(\lambda^n)}{\lambda^n},\end{aligned}\tag{6}$$

$\mu(\lambda^n)$ the correspond $\sum_{x\lambda^{-n}\leqslant p\leqslant x\lambda^{1-n}} 1/p$.

This (6) is the Riemann formula to convert theorem. Or: the distribution of prime numbers Fundamental Theorem. For example: Set $a\leqslant \ln x/\ln\lambda$, $\lambda=x^{1/\sqrt{x}}, x=256$ to ge λ =1.41421356, a=16, By (6) control: $s(256)-s(16)+\pi(16)=53.771491004252822$, Actual: $\pi(256=54)$, Small x the method of calculation is very close to $\pi(x)$. Great x, the method of calculation is accurate. For example: Set $a\leqslant \ln x/\ln\lambda, \lambda=x^{1/\sqrt{x}}$ by (6) control:

x,	$\pi(10^x)$,	$s(10^x)-s(\sqrt{10^x})+\pi(\sqrt{10^x})$,
8,	5761455,	5761455,
9,	50847534,	50847534,
10,	455052511,	455052511,
11,	4118054813,	4118054813,
12,	37607912018,	37607912018,
13,	346065536839,	346065536839,
14,	3204941750802,	3204941750802,
15,	29844570422669,	29844570422669,
16,	279238341033925,	279238341033925.

Although there is no detailed reasoning is given, however, accurate primes distribution function, can trust Riermann conjecture is established. Set $s>1$, By (6) get

$$\pi(x^{1/s},x^{1/2s})=s(x^{1/s},x^{1/2s})\tag{7}$$

By (7) get:

$$\pi(x)=s(x)+c,(x\to\infty)\tag{8}$$

Here (8), and c is a small finite number, is substantially equal.

6 the conversion theorem inequality

$$s(x)-s(\sqrt{x})+\pi(\sqrt{x})-\pi(\sqrt{x})<\pi(x)<s(x)-s(\sqrt{x})+\pi(\sqrt{x})-\pi(\sqrt{x})\tag{9}$$

By (9) get:

$$s(x)-s(\sqrt{x})<\pi(x)<s(x)-s(\sqrt{x})+2\pi(\sqrt{x})\tag{10}$$

By (8) to confirm:

$$s(\sqrt{x}) < 2\pi(\sqrt{x}) \tag{11}$$

By (10), (11) get:

$$s(x) - 2\pi(\sqrt{x}) < \pi(x) < s(x) + 2\pi(\sqrt{x}) \tag{12}$$

By (12) Can be converted to the form of computing. Riemann conversion calculation of the theorem Set $x \to \infty$, by (4), Ignore $c(x)$, by $u(x/\lambda^n)$ get:

$$\sum_{x\lambda^{-n} \leqslant p \leqslant x\lambda^{1-n}} 1/p = \ln\ln(x\lambda^{1-n}) - \ln\ln(x\lambda^{-n} = \ln\frac{\ln(x\lambda^{1-n})}{\ln(x\lambda^{-n})} = \frac{\ln\lambda}{\ln x - \ln\lambda^n} \tag{13}$$

By (13) On behalf of (6), get:

$$s(x) = x(\lambda - 1)\Sigma_{n=1}^{a}\frac{\lambda^{-n}}{\ln x + \ln(\lambda^{-n})},\ (x \to \infty). \tag{14}$$

Set $\lambda \to 1$ by (14), get:

$$\frac{1}{\ln x - \ln\lambda^n} = \frac{1}{\ln x}, \tag{15}$$

By (15) Prove Gauss guess. By (5), get:

$$Li(x) = s(x). \tag{16}$$

This (16) is fundamentally the same. For example: Set By (5), (14) control.

x,	$\pi(10^x)$	$Li(10^x)$	$s(10^x)$
1,	4,	6,	5,
2,	25	30,	30,
3,	168,	177,	177,
4,	1229,	1246,	1246,
5,	9592,	9630,	9630,
6,	78498,	78628,	78628,
7,	664579,	664918,	664918,
8,	5761455,	5762209,	5762209,

7 prove the Riemann Hypothesis

In 1901, Von Koch Proof: If Riemann conjecture is true, then the [6]

$$\pi(x) = Li(x) + O(x1/2\ln x) \tag{17}$$

Turn, If we can prove that (17) holds, then the Riemann conjecture certainly holds. From (12), (16) obtained:

$$Li(x) - 2\pi(\sqrt{x}) < \pi(x) < Li(x) + 2\pi(\sqrt{x}). \tag{18}$$

Easily confirmed by (18) (17). Riemann's conjecture. Discussion: The mathematician said Goldbach Conjecture access emergence of mathematical ideas, new methods must be accompanied by proof of the

Goldbach conjecture. The Riemann conjecture must be accompanied by a change in the traditional understanding of mathematics. Indeed. Generalized Riemann prime number theorem:

$$\pi(x^2) = r(x^2),$$

$$r(x^2) = \frac{1}{2}\sum_{n=2}^{x} \frac{(2n-1)\mu(n^2)}{\ln n - \ln(n-1)},$$

Let $x = 8$, computing $r(64) = 18.0753582157848046$, the actual $\pi(54) = 18$, is extremely accurate, extremely easily understood. Proof of Gauss's guess, vaguely see the distribution of prime numbers secret doorways. And the discovery of the generalized Riemann prime number theorem, completely open the secret doorway to the distribution of prime numbers. By generalized Riemann prime number theorem get:

$$r(x^2) = \sum_{n=1}^{x} \frac{n - 1/2}{\ln n - 1/2},$$

Calculate:

x	$\pi(x^2)$,	$Li(x^2)$,	$s(x^2)$,
16,	4,	60,	60,
32,	25	181,	181,
64,	168,	577,	577,
128,	1229,	1920,	1920,
256,	9592,	6594,	6594,
512,	78498,	3069,	3069,
1024,	664579,	82138,	82138,
2048,	5761455,	296114,	296114.

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